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STABILITY AND CONTROL STUDY OF A SMALL ROCKET LIFT DEVICE

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I. SUMMARY

A stability and control study of a Small Rocket Lift Device was performed employing analytic methods. Mathematical definition of the operator's control actuation logic and visual orientation sensing capabilities were formulated. These, together with the equations of motion of the system, were programmed for an IBM 704 digital computer and pitch plane trajectories were computed. Additionally, a yaw plane trajectory and a one degree of freedom roll dynamics program were run on an IBM 610 digital computer.

The operator was assumed to apprise himself of his orientation through visual reference to the ground. Thrust direction and magnitude were controlled by the manipulation of hand levers. The linkage of these controls was varied to determine the effect on stability and control. It was determined that controllable flight could be achieved for the man-vehicle system studied.

The effects of CG shift, wind, a severe body contortion, variation of flight rules, and temporary loss of thrust were investigated. to pg. 2

II. INTRODUCTION

This report describes a stability and control study performed as a part of a feasibility study of a "Small Rocket Lift Device" (SRLD) performed by the Aerojet Systems Division for the U. S. Army Transportation Research and Engineering Command under Contract No. DA44-177-TC-595. Certain weight and dimensional and motor performance data defining the device was extracted from other portions of this study. Additionally, a Human Engineering Study performed under this contract contributed information concerning the physical characteristics of the operator of the device (Reference 1).

Since the objective of the study was to determine the feasibility of a device of this type rather than to precisely predict the optimum vehicle performance, this stability and control study is directed toward determining whether a system configuration may be defined which allows safe, controllable flight. It was felt that optimizing the operator's actions was neither necessary nor desirable. The computed flights, therefore, do not show either the performance of an optimum system or the optimum performance of the selected system. It will be found that this report departs somewhat from the classical form of reporting in which the method of approach is first presented independent of the results. This was found desirable since this study involved both the determination of a method of analysis, and the generation of a satisfactory vehicle geometry. The capabilities of the vehicle and the operator, which might, in part, more properly be considered results, influence the form which the method of analysis must take. It was therefore considered advisable to introduce, along with each point of the development of the method of analysis, that item of vehicle or operator capability which determined the form of that portion of the analysis. It is felt that this form of presentation will be the most meaningful.

Since the objective of the basic study was to investigate the overall feasibility of a SRLD, other factors besides stability and control had to be considered in establishing a system configuration. Thus, this stability and control study was directed toward investigating a general configuration which satisfied the requirements of low cost and weight, compactness, reliability, and performance potential.

The configuration which was considered consists of two pressure-fed liquid monopropellant motors mounted to a back pack which supports all vehicle components. A single throttle controls the propellant flow to both motors while a thrust differential between the motors is effected by employing a flow dividing valve. Thrust vectoring is effected through both gimballing of the nozzles and the use of a jetavator. Extensions of the pack structure on each side of the operator bring the hand controls to a position convenient to the operator.

III. DISCUSSION

A. OPERATOR CAPABILITY

In order to perform a quantitative study of the dynamics of a guided airborne system, it is necessary to define, in mathematical language, the guidance and control systems. In the case of the small rocket lift device, the operator performs the guidance and control actuation functions. It is apparent that the complexity of the entire study is largely a function of the complexity of the equations defining the operator since the equations of motion are for the most part invariant from system to system. For this reason, considerable attention was given to the operator characteristics before a method of solution was selected.

One of the first choices which had to be made was between a skilled and a relatively untrained operator. Obviously, the less skill assumed on the part of the operator, the more conservative the results of the study become. Additionally, it was felt that a study which assumed a minimum of operator capability would be the most productive in pinpointing the areas in which preflight training was required and indicating the degree of proficiency which would be acceptable for safe free flight operation. For these reasons, it was decided that an unskilled operator would be used as the mathematical model.

In order that the operator's response pattern be defined, it becomes necessary to resort to analogy. Fortunately, a close analogy to this type of vehicle exists in the automobile, for which the behavior pattern of an unskilled operator is well known. Additionally, the frequency of control manipulations is of the same order of magnitude as would be expected for the small rocket lift device. One of the criteria which specifically defines the form which the solution must take is the degree to which the operator anticipates future events. To determine this, consideration is given to the analogy. "Learning the brakes" is one of the first accomplishments of the student driver, and this is effected quite rapidly. The application of pressure to the brake control is far from smooth and no attempt is made to "feather" the control, but the judgement of the point at which braking action should be initiated in order to stop at a further selected point is quite good after only a few tries. The assumption has, therefore, been made that the operator of the SRLD is capable of anticipating his stopping point for a braking maneuver executed at some position and velocity. The transition time to move from the accelerating to the braking configuration has been assumed not to enter into his calculations. Certain maneuvers which the operator must perform will have to be made familiar to him during a training period in some sort of a tether rig. Among these is the braking maneuver. As in the case of the student driver, the potential SRLD operator will have to learn the brakes. This must be done for both horizontal and vertical flight.

Other than the braking distance estimation, the student driver does not appear to anticipate to any great degree, but rather operates the controls until conscious of a condition of overcorrection, at which time he institutes a control reversal. (It should be pointed out here that the absence of wildly erratic steering operation should not be taken as an indication that feel is immediately developed upon first encountering a control. Rather, it appears to be an indication that the student has developed some feel through observation of more experienced operators of the vehicle over a period of time. The control of the gas and clutch pedals is a much better example of this over-control effect since the student is much less aware of the experienced operator's utilisation of these controls, they being much less visible than the steering wheel). The assumption was made that a similar condition existed for the operator in this study. There is some reason to believe that an operator would gain some feel for the system even in the short time consumed by a single flight. However, in order to keep this study conservative, it was assumed that no feel developed, and that the operator would either apply the maximum possible amount of force to the controls or release them. There are two considerations involved in arriving at this type of response pattern. One is that if the operator has no feel for the controls, a response pattern must be introduced to him which does not involve gain. The other consideration is that the inexperienced operator is liable to panic, and attention should be given to the mode of operation which would be likely in this condition. One response pattern which does not require feel for the entire system, which may be introduced to the operator, is the application of a constant level of force to the controls in response to a sensible error. Obviously, any force level except maximum would require some practice to develop a feel for the control by itself, although free flight experience would not be necessary for this.

It might be expected that an inexperienced operator, if he were to decide he was losing control, would panic; and in this state would apply the maximum possible force to the controls to attempt to correct an increasing error. In light of these considerations, it appears that the SRLD should be designed so that satisfactory flights may be performed when the maximum possible control forces are applied in response to sensible errors, and that the operator be trained to operate the vehicle, initially, in this manner. The force which the operator applies was assumed to be equal to the force which 95% of all potential operators would be capable of applying.

The author has observed, for the purpose of testing the validity of the description of the student driver: automobile control response pattern, several drivers, both experienced and inexperienced, in several automobiles with different control characteristics. The observed performance indicates that the description is valid.

B. RESPONSE PATTERN

If feel were developed, the operator might be expected to apply a control force which was linearly proportional to the error being corrected (Reference 2). If we define the position error as Ω, then we can define the control moment which would be exerted as $M = c_0\Omega + c_1\dot{\Omega} + c_2\ddot{\Omega}$.

C_2 would probably be negligible in any case and would be ignored in a study including feel (Reference 2). Considering only one term of the right member ($C_0\Omega$ is chosen for this example) we would show a variation of moment with Ω as shown in Figure 1a. (C_0 is assumed negative). Since the operator is limited in the size of the moment he can apply, the M vs curve must be modified as shown in Figure 1b. Additionally, there is a threshold level of error, Ω. , below which the operator would not be conscious of an error, and therefore would apply no moment. This results in a response curve of the form shown in Figure 1c (these figures are not to scale). An analogous development would determine the response curve for the $\dot{\Omega}$ term. The unfortunate part of adding this elaboration to the study is that we cannot, with any degree of confidence, assign values to C_0 and C_1, the gains. One could determine the values which most enhanced the computed performance of the system but these would be optimum values and it would be overly optimistic to assume that the operator would develop optimum conditioned response to the system at a rapid enough rate to satisfy safety requirements. It is possible, however, to assume that the operator is instructed to apply the maximum possible force to the controls as soon as an error is perceived. While this will lead to an over-control condition, the type of flight path which results may be predicted and control geometry chosen to allow safe flight within this framework. This approach was taken in this study. If a successful flight, primarily a safe one, may be accomplished this way, then any "feel" developed would only enhance the performance. The control response for a "no skill" type of system would be that shown in Figure 1d.

The question will naturally arise as to how different the flight predicted by the "no feel" system will be from that expected from a system in which the operator has developed a certain amount of feel. This must be answered in two parts:

If the flight is completely "on design" it is to be expected that the two systems will differ greatly since the oscillations about correct attitude values which are expected in a system with an unskilled operator would be slight in a system in which the operator has perfect feel. However, it may be seen that in a system in which errors were liable to increase rapidly to a magnitude requiring maximum control force, the integrated control moment over a time T, considering gain, $\int_0^T M\,dt = \int_0^{t_{M=M_{MAX}}} (c_0 \alpha + c_1 \dot{\alpha})\,dt + M_{MAX}(T - t_{M=M_{MAX}})$ is not too different from $M_{max}T$ once the error is introduced, and the dynamics of the two cases is quite similar.

(The increase of the error, after the correction is initiated, is large compared to that during the reaction time). This would indicate that once a large attitude or rate error had occurred, the behavior of the system with feel would, at least for a portion of the flight, be quite similar to the behavior of the system as herein defined. It may be mentioned that a linear representation is valid only for a system with random input, since the experienced operator would otherwise anticipate future errors.

C. VEHICLE GEOMETRY

A general vehicle geometry must, of course, be defined in order that performance and stability may be determined. The first obvious requirement is that the operator must be capable of leaving the ground. This may be accomplished in two ways if rockets are employed. The first is to provide some vertical thrust component which is smaller than the operator's weight so that when he jumps up, his time off the ground is increased and he may translate horizontally while free of ground contact. The disadvantage of this system is that the operator cannot to any great degree control his vertical position and may return to the ground at a rather disadvantageous position. The other possibility is to provide a vertical thrust component which is somewhat larger than the operator's weight so that he may control his time off the ground within the limits of the duration of his propellant supply. This system promises both increased performance and improved safety and is the one considered in this study. It is felt that a prototype vehicle which is basically what is considered herein should have a thrust not too much greater than the operator's weight, since a vehicle with high levels of upward acceleration available might "run away" with an inexperienced operator. The maximum thrust of the vehicle which was considered in this study is taken to be 1.11 times as great as the takeoff weight, however, the effect of increases in thrust on performance has been investigated, and shown promise of considerably enhancing performance. The takeoff weight of the man-SRLD system considered herein is 280 pounds including 45 pounds of propellant.

Since, with a low thrust to weight ratio, climbing performance will be quite poor, it may be expected that the system will operate in close proximity to the ground. High downward accelerations are therefore neither desirable nor necessary for flexible performance. Also, it is desirable to keep the thrust level above the point at which unstable engine operation might occur. Since the inability of the operator to obtain thrust when desired might have dangerous consequences, it is felt inadvisable to attempt restarts in the air since a large portion of motor failures occur at ignition. Additionally, since, as will be discussed later in more detail, the motors which provide the vertical thrust components are also employed to provide other of the required forces and moments, it is considered desirable that the ratio of full thrust to idle thrust not be too great. Since all of these considerations indicate the desirability of a fairly high idle thrust, a ratio of full to idle thrust of 2:1 was selected. The computed performance results showed this to be a satisfactory ratio. The linkage controlling the thrust was designed and the value of idle thrust as yielded by this design was 157 pounds, compared to a maximum value of 310 pounds. (A propellant load of 45 pounds and an Isp of 100 seconds were assumed in this study).

The attitude which the operator assumed was next considered. It was felt that the operator would be the most at ease in a head-up attitude, so the performance requirements were considered to determine whether the system could be so constructed that this attitude was feasible. It was determined that the drag at 60 miles per hour, which speed the operator would certainly not wish to exceed, was only 62 pounds for the operator flying forward and essentially upright. Since the thrust deflection angle required to develop this value in horizontal thrust (with a total thrust of 310 pounds) is only some eleven and a half degrees, there is certainly no reason to require larger pitch angles from this standpoint. The horizontal acceleration corresponding to this thrust level, at takeoff weight, is 7.13 ft per sec^2 which the results have indicated is quite adequate. The legs would hang down below the operator, since restraining them in the sitting position would require additional system weight and would only deteriorate their value as shock absorbers during landing. Additionally, as was determined during the computer study, some motion of the legs will be required during the flight to trim out the CG shift as propellant is utilized, or the fore and aft pitch control torques become markedly asymmetric. (This will have to be a trained response, but this should be possible before free flight is attempted).

It is now apparent that the thrust axis, when undeflected, should be parallel to the vertical centerline of the operator. To prevent the thrust from exerting a torque about the CG, in the undeflected position, the thrust axis must, of course, pass through the CG. Since the tankage and the bulk of the other system components are on the operators back, the combined CG of the system is somewhat above the operators waist and fairly far back, so that if a single motor were mounted below the back pack, the thrust axis might be arranged to pass through the system CG. The alternative of placing a single motor above the operator's head is neglected for obvious reasons. The objection to employing a single motor below the pack is the difficulty of, through a simple device, obtaining a torque about the vertical axis to induce a yawing motion without inducing roll and pitch moments. The next alternative is to supply two motors arranged either fore and aft or side by side. The side by side disposition is chosen since the major expected motions of the operator may be expected in the plane containing the operators vertical axis of symmetry and his roll axis and arranging the motors symmetrically about this plane allows the use of motor gimballing, in a direction in which there is the le st possibility of the exhaust impinging upon the operator.

To determine the form the remainder of the system must take, consideration was given to the flight plan which the operator is to be required to follow.

D. RELATIVE MAGNITUDE OF VARIOUS MOTIONS

Previous studies (See section on Human Engineering, Ref. 1) have shown that the time required for an operator to respond to a stimulus is proportional to the number of different pieces of data which make up the stimulus, and the number of different controls which the operator must make a decision to manipulate in response to the stimulus. It is thus apparent that a vehicle which operated in a two dimensional format would require somewhat less acuity of reaction on the part of the operator than one which moved in three dimensions. Unfortunately, there is no way of restraining a free flight vehicle to move only in two dimensions. It is possible, however, to reduce the complexity of the decisions which the operator is required to make by specifying flight plans for training which require maneuvers to be performed in what is basically a plane, with all other control motions used only for the purpose of restraining the flight to this plane. The operator, then, is relieved of the requirement of executing and coordinating several simultaneous maneuvers. Considering the flight time which is attainable with this system (approximately 17 or 18 seconds as a maximum), it may be seen that the number of maneuver decisions which the operator must make is quite high compared to the flight time, and if he should get himself into a combination of attitude, position, and velocities which are incompatible with safe landing, he may well find that he has run out of propellant before he is able to sort out and correct the multitude of errors.

The mission which it is most desired for the SRLD to perform is what is basically a long jump. This maneuver may be carried out, in the absence of any perturbations of the design flight, entirely in the pitch plane. The controls which are required for flight in this plane are a throttlable motor which has been previously discussed plus a control which may be used to control pitch orientation. As mentioned, two motors are provided which yield a thrust somewhat greater than the loaded weight of the system parallel to the vertical axis of the operator. A horizontal thrust component which is sufficient for flight requirements could be obtained by pitching the operator and the thrust vector forward several degrees. Additional use may be made of these motors if they are so mounted that it is possible to rotate them so that a pitching torque may be produced. The reaction of an operator, as has been stated previously, will vary with the complexity of the correlation of stimulus and control motion. Consideration has been given to this in selecting the proper axis of the motors.

E. POSITION OF MOTOR GIMBAL AXIS

It was determined that the reaction time that could be expected in a situation where the type of reaction and the type of stimulus were known and correlated, but the sense of the stimulus and its time of occurence were unanticipated, would be approximately 1/4 second for the average unskilled operator. (A value of either .26 or .25 was used for convenience in the various computer routines to make it an even multiple of the computing interval.) If a vehicle is to be operated through the manipulation of a series of controls, then the correlation of error direction and required control deflection may be greatly simplified, and the speed of control response improved correspondingly if for all cases of error, the direction in which the control is to be moved is the same as that in which the control force is desired. Considering first the control of horizontal velocity and position, it is apparent that if the operator desires to go forward, he is required to provide a forward thrust component. For the case of a vehicle which is operated so that an initially vertical axis remains essentially vertical throughout the flight, with the motors so disposed that when at rest, they are parallel to the vertical axis, the required forward thrust component would be obtained by rotating the thrust axis of the motors so that the direction of the thrust axes was up and forward (the upward thrust component, naturally, counteracts the pull of gravity). It is apparent that if the control linkage is so arranged that the thrust axis is always parallel to the control lever, the requirement that the motion of the control be in the same direction as the desired motion of the vehicle (in this case forward) is satisfied. If the axis about which the motors pivot passed directly through the CG of the system, then no torque would be exerted about this CG and there would be no change in the pitch inclination of the system. Unfortunately, the CG is bound to shift as propellant is expended, and other perturbations such as aerodynamic forces and motions by the operator are bound to induce upsetting moments which will result in changes of pitch orientation. Accordingly, it becomes necessary to provide a means of supplying a restoring torque. The motors which have already been made available to provide horizontal and vertical thrust components may be used for this purpose if the pivot axis is displaced above or below the CG. If the motor pivot were displaced below the CG, then pivoting the thrust vector from straight up to up and forward would cause the system to pitch backward. This, it may be seen does not satisfy our requirement that the motion of the control system be in the same direction as the required motion of the vehicle. This requirement may be satisfied by displacing the pivot axis of the motors above the CG, which is the geometry of the vehicle herein considered.

Several additional considerations are involved in the determination of the position of the pivot axis. If the pivot axis is placed too high, sensitivity is too high and the limits of motor deflection must be kept small whereupon manufacturing tolerances become increasingly significant. If the motor axis is placed too low, then the arc swept by the thrust vector (with a reasonable deflection) between the stops may not always include the CG as it moves with propellant depletion. Although the operator will probably be required to move his legs to trim out the CG, it is still felt that it is desirable to so dispose the motors that pitching torques in both directions are always available even without this CG trimming.

It was found that spacing the motors 15 inches to either side of the vertical axis of the operator would keep him free of the exhaust blast and still allow a compact design. This value was used in these computations.

F. ROLL, YAW AND LATERAL DISPLACEMENT

The motions which still remain, and for which corresponding controls must be considered, are yaw, roll, and lateral displacement. Since, as was discussed previously, flight maneuver requirements would be limited primarily to the pitch plane, it is not necessary to provide controls intended, primarily, for effecting large translations in the lateral direction. Our concern will be primarily with effecting roll and yaw stability. The controls provided for this purpose allow the lateral displacement to be controlled but it should be understood that the control of this motion will basically be limited to keeping the lateral displacement small.

G. CRITERIA FOR ROLL AND YAW STABILITY

The most likely source of a disturbance which would cause a lateral shift is a cross wind. This could be corrected for by holding a small roll angle so as to provide an upwind thrust component. It is felt that roll stability may be demonstrated if the vehicle may be shown capable of performing a change in roll orientation with no unduly large oscillations. An investigation of this control requirement was made with a one degree of freedom analysis employing an IBM 610 digital computer. The control moment was assumed to be provided by a differential thrust between the two motors with a maximum value of 10 pounds at full thrust. The other alternative for holding the lateral translation to a low level would be to yaw the vehicle into the wind. It was decided that if the vehicle could be flown in the presence of a crosswind, with the heading of the vehicle adjusted so that the operator always faced his objective, and it could be shown that no large heading errors developed, then it could be concluded that satisfactory yaw control existed. It must be reiterated that this and the roll study are not directed primarily at determining the vehicles applicability in controlling lateral displacement, but rather to determine the systems roll and yaw stability when controls affecting these items are used to correct a perturbation to the pitch plane flight. Yawing moments are produced in the system considered by fixing a jetevator to one of the motors. This jetavator may be pitched fore and aft, and will produce a maximum of two pounds of thrust normal to the motor axis at full thrust. Yaw performance was studied in a two dimensional trajectory which was run on the IBM 610. Motion was confined to a horizontal plane.

In keeping with the concept of making the direction of the control movements as natural as possible, the roll control was so designed as to cause the vehicle to lean in the direction in which the stick was tilted and the yaw control made to rotate about a vertical axis so that the vehicle would rotate in the direction in which the control was twisted.

H. SUITABILITY OF TWO DIMENSIONAL STUDY

A free-flight vehicle is, naturally, capable of six modes of motion translating along and rotating about three orthogonal body centered axes. Since a two dimensional analysis was employed in the principal part of this study, it seems desirable to show justification for this method of approach. If a vehicle is intended to perform maneuvers corresponding to motion in all six modes, then a three dimensional study would be required. However, if, as in the case of the system under study, flights may be made with maneuvers required in only a few of the possible modes of motion, a study may be set up which involved only those modes, with accessory studies made of stability in the other modes. The basis for this is the consideration that if controllability in the directions other than those involved in the intended maneuvers is good, then motion in these non-maneuvering directions is quite limited and has only a minimal effect on the performance of the intended maneuvers. The primary object of the system being considered being to allow the operator to jump over obstacles, the flight plan which is of primary interest is one which lies completely in the pitch plane. Accordingly, a two dimensional analysis has been evolved to mathematically represent flight in the pitch plane. This study has been programmed and run on an IBM 704 digital computer. The development of the equations of motion is quite lengthy and has been included separately in the appendix. The equations of motion employed for the yaw and roll studies, being basically simplifications of the pitch plane equations of motion are not detailed herein.

I. OPERATOR LOGIC IN THE PITCH PLANE

Considering flight in the pitch plane, we find six pieces of data which are of concern to the operator: vertical position, vertical velocity, horizontal position, horizontal velocity, pitch inclination, and pitch rate. It may be expected that any one of these or any combination of these items may, at any time during the flight, differ from the values which the operator desires to maintain. If only a single one of these items varies, the correction to be made is obvious, both to the operator and the analyst. Unfortunately, it must be expected that several of these errors will require correction at once. The corrections required by these several simultaneous errors may be incompatible. Therefore, in performing this type of study, it is required that predictions be made as to the order of importance assigned by an actual operator to the several concurrent errors.

1. Operator Safety

One basic assumption which may be made is that the operator will be more concerned with personal safety than with satisfying the mission requirements. The mission requirements concern, basically, only a change in position. Safety requirements may be though of as being concerned with rates as a function of position. It may be expected that the primary concern of an actual operator would be to keep his downward velocity to a safe level. In this study, a value of 20 feet per second has been chosen. Considering the weight of the pack, this velocity, which is equal to approximately a 5 foot free fall, seems a good approximation of the maximum safe impact velocity. Since he could decelerate while descending, this leaves the operator with some safety margin if this velocity is attained at a distance above the ground. As will be seen later, the free flight altitude range for the system chosen is quite limited, so that other factors affect the vertical velocity before the design limit speed is reached. It should be mentioned that a logic branch exists at the beginning of the computer simulation of the operator's logic which unconditionally instructs him to cut back thrust if he pitches more than 90° from vertical so that the downward acceleration is minimized. This will also need to be a learned response on the part of the actual operator.

It would at first appear that the operator might be more concerned with pitch orientation than with pitch rate. However, it will be seen from the results of the study that the angular accelerations which must be available to the operator in order to perform a mission are so high that the operator must learn to respond to pitch rate before pitch orientation in order that the pitch oscillations of the system not be divergent. Since it may be predicted that the operators would, of himself, consider remaining upright as his primary safety consideration (except for the maximum downward velocity consideration, which, as mentioned earlier, does not enter for a vehicle with the performance of that considered here, and the compulsory thrust reduction for pitch angles over 90° in which case the operator is in uncorrectable trouble anyway), we may consider that the learned pitch rate response will be the primary control reaction both during an actual flight and in this study.

A maximum upward velocity should be specified, since duration is short and the operator should not be put in the position of rising so fast that he is unable to halt his upward velocity and return to the ground before his propellant is expended. A maximum upward velocity of 20 feet per second was chosen for this study. It was found, however, that the system capability is such that excessive upward velocity will not be a problem for flights across fairly level ground.

A maximum horizontal velocity should be specified, again to avoid placing the operator in the position of having insufficient propellant to decelerate to a safe landing speed. A value of 30 feet per second was selected for this study, and this appears to be a reasonable value for test purposes.

The safety considerations mentioned may be represented by the logic network of Figure 2. This figure diagrams only those tests which the operator would make in the interest of flight safety in the pitch plane.

2. Mission Requirements

Next to be considered are the tests which the operator must make to satisfy mission requirements. As mentioned previously, these concern mainly changes in position. An attempt has been made to write the program in such a way that the flight path which is computed duplicates as closely as possible that which would be followed by the actual operator. This involves the introduction of maneuver commands in the program at such a time as the equivalent thought might be expected to come to the actual operator. This process is illustrated in Figure 3. Assuming that the operator has selected a target point and a cruise altitude, his first decision would be to rise from the ground. This decision is represented by setting the first required altitude (h_{ro}) to the cruise altitude and the first required horizontal displacement (x_{ro}) to zero. After the operator has risen several feet (to altitude hc), he would make the decision to translate toward the target point. This is represented by setting x_r equal to x_{r1}, the range of the target from the takeoff point. When the operator receives the information that his x position is in error, he sets his required pitch orientation such that he will translate toward the target. For a thrust of 310 pounds, and a weight of 280 pounds, it is found that the thrust axis may be pitched 25.4° from the vertical and the vertical component of the thrust will be equal to the weight. It is considered desirable that the thrust axis inclination in flight does not exceed this value. Since a maximum pitch orientation error of 15° was set as an achievable goal for the vehicle, a design pitch angle for cruise of 10° was arrived at. Thus, when the operator received the information that x_r is considerably greater than his present position, x, he sets ϕ_r, his required pitch angle, to +10°, and attempts to hold this angle.

J. ERROR JUDGMENT

At this point, some discussion apropos to threshold values of error sensing, and judgment accuracy is in order. Tests of several individuals indicated that over distances on the order of 10 to 100 feet, the distance could be estimated within approximately 20%. This figure is conservative since distances were estimated as a number of feet which involved estimation of both the distance and the length of a foot. However, it was decided that this conservative figure would be used in preference to data presented in the literature which indicated extremely small errors in distance estimation under carefully controlled laboratory conditions. It was felt that it would be unrealistic to employ this data in a synthesis of actual flight conditions. This was taken to indicate that if an operator with only visual reference to a textured ground plane attempted to hold a constant altitude, he would become conscious of an altitude error when he had departed by 20% from the desired altitude. In Figure 3 this is indicated by the two lines $h_{r_0} + K_1h$ and $h_{r_0} - K_1h$ which bracket the desired altitude h_{r_0}. Thus, we would expect the operator as long as he was between the two bracketing lines, would consider himself to be at altitude h_{r_0}. Several further extrapolations have been made from this data to generate other portions of the mathematical definition of the operator. Though these extrapolations are largely intuitive, it is felt that they are of sufficient accuracy to be useful in this study. One extrapolation is that the operator is conscious of not being directly over a chosen point when his x distance from the point is greater than 20% of his altitude above the point in question. The other extrapolation is that when the operator is some distance from a target point, his error in the judgment of horizontal range is equal to 20% of the slant range to the target. Although some data was found which indicated that smaller values would be detectible, it is assumed, to be conservative that the operator was conscious of motion when his velocity exceeded 0.2 feet per second per foot distance from his reference point.

In developing the analog of the braking distance estimation, several assumptions were made.

First, it was assumed that the operator had been trained in a tethered rig so that he was conscious of the horizontal deceleration rate of the vehicle when pitched back to the design angle of 10°, and with the thrust at both the maximum and idle levels. He was also assumed to be familiar with the vertical deceleration rates, both when an upward velocity was being reduced with the thrust cut back to idle, and when a downward velocity was being reduced by applying full thrust.

Secondly, the assumption was made that the operator, in the event that the thrust was at neither the maximum or minimum setting would make his braking estimations on the basis of whichever one of these stops the throttle was being moved toward since he would expect that the thrust would be at that level in a fraction of a second. If the throttle were being held in a constant position, it was assumed that the existant thrust would be used in these computations.

Also assumed was that the operator knew his velocity exactly and the distance to the point at which he intended to stop to an accuracy of 20% of the slant range. The operator was assumed to over-estimate the distance which made the study conservative since it was expected that reversing direction would require more propellant than correcting an undershoot. The operator was also assumed to know his vertical range to the desired altitude within 20% of his own altitude, the error being again on the overestimation side.

K. DEPARTURE OF PREDICTED FROM ACTUAL PERFORMANCE

It may be seen that the possibility exists that considerable accuracy in predicting actual flight paths was lost through this series of assumptions; however, the prediction of vehicle stability is not affected since this is concerned primarily with attitude and angular rates. The trajectories determined differed from the expected actual trajectories mainly in the size of the overshoot predicted upon landing. The interpretation to be placed upon the results ingageing horizontal range capability is that they actually represented attempted flights over somewhat different distances than those specified in the computer problems. Since none of the assumptions appear greatly unrealistic, it is recommended that the target missions for prototype vehicles be held within the range of the computed trajectories until considerable free flight data has been obtained so that comparison may be made with the computed results. It should be borne in mind that the test pilot will have to be given instructions and ground training in the operation of the vehicle, and if the training is such that the operator will perform the horizontal braking maneuver using the rules assumed for the study, then the actual flight will be quite similar to the computed flight. The difference will come in when the operator is allowed to deviate from the rigid set of flight rules. This, being a rather important point, should be discussed in some detail.

It is assumed that the operator will attempt to brake his forward velocity be pitching back to the design angle of -10° and attempting to hold this angle, Since the operator will, concurrent with the horizontal braking, start his descent, an actual operator will in all likelihood determine that he may increase his horizontal braking by pitching further backward. The reason for the design pitch angle as selected of 10° was to make available to the operator at all times a vertical thrust component greater than his weight. An actual operator, reasoning that he requires for a period of time a smaller amount of vertical thrust and desiring to increase his horizontal deceleration will be sorely tempted to temporarily increase his backward pitch. This would obviously make a greater horizontal thrust component available and decrease the overshoot.

It has been mentioned that a threshold value for sensing the presence of a velocity was defined. The object of this is to include the analog of a pilot judgement to the effect that if it appears to the operator that his position is correct, then, when he senses a velocity tending to carry him away from this position he will apply the appropriate control force to resist this motion.

L. CRUISE ALTITUDE

Returning to the description of the flight plan, the operator has selected a pitch angle of +10 degrees for cruise and attempts to hold this value. Experiment has shown that a man can sense a deviation from the vertical of as little as 0.8 degrees. To be conservative, it was assumed that the operator was able to sense a 3° or greater deviation from his desired pitch orientation. He would therefore react to correct his pitch angle when it became greater than 13° or less than 7°. As mentioned previously, the operator must be trained to respond to pitch rate before pitch angle, and it was assumed for this study that the operator allowed a maximum pitch rate of 10° per second. Higher values were investigated but performance deteriorated rapidly as the allowable pitch rate was increased.

The operator will continue to accelerate forward, holding his pitch angle near 10° and his altitude near the desired value until he either reaches the maximum allowable cruise velocity or the point at which he elects to initiate braking to stop above the target point. If he reaches his maximum cruise velocity, he will set the desired pitch angle to 0°, and if he exceeds it to -10°. If he elects to commence braking he will set the desired pitch angle to -10°. Upon reaching a point which the operator considers to be above the target, h_r, the required altitude is set to zero.

The operator now attempts to stay, within his ability to judge, above the target point as he descends.

M. LANDING ROUTINE

The operator's logic in effecting a landing is depicted in Figure 4. Assuming that the vertical velocity is downward but does not exceed the allowable downward velocity, $|V_{Y_D}|$, the operator tests whether he desires to land (i.e. $H_R = 0$?). If he does, he tests whether his horizontal velocity (V_x) is excessive. The allowable horizontal speed when landing (V_ℓ) was set equal to the speed which it was expected the operator could run at: $V_{\ell\ FORWARD}$ = 20 feet per second, $V_{\ell\ AFT}$ = 10 feet per second. Since the study considered safety as more important than performance, it was felt that the conservative sense of the estimation of velocity would be the underestimation of velocity. Accordingly, the test is whether $V_{\ell\ AFT} - K_5 h < V_x < V_{\ell\ FORWARD} + K_5 h$, where K_5 is equal to 0.2 ft/sec/ft altitude and is the same coefficient as that used in the earlier test of whether any velocity was apparent. If the horizontal velocity is excessive, the operator increases thrust to postpone the landing. If the horizontal velocity is within the allowable range, the operator tests to see if he is within the specified horizontal distance from the desired landing point. This distance is defined as ΔX_A in Figure 4, and the judgment tolerance, $K_4 h$ is applied to it. If the operator determines that he is not within the range $|X_A - X| < \Delta X_A + K_4 h$, he increases thrust to forestall landing. If it is determined that he is within the range, he next considers his pitch orientation and pitch rate. If they are in opposite directions, his pitch conditions are judged satisfactory for landing and the approach to the ground may be continued. If Ø and Ø are not in opposite directions, the next step in the program is to test whether both are zero. (This test is necessary to the computer logic, at this point in the program. The order of decisions is somewhat different from that in which they would be made by an actual operator, but the end result is the same). If both Ø and Ø are zero, the descent may be continued. If the value of either is not zero, it is tested to see if it exceeds the allowable landing values (a pitch orientation of 1/2 radian or less was assumed allowable at landing, the test being whether $|Ø| > Ø_A$). The allowable pitch rate at landing, $Ø_A$, was assumed to be 1 radian per second. Both of these values are larger than those allowable in flight, but they will occur during control transients. If these tests are reached, both Ø and Ø are in the same direction, or one of them is zero. In either case the required direction of control deflection for one would be the same as that for the other if it is non-zero. If these tests indicate that a correction is required, the operator tests whether the pitch control at the time is deflected in the required direction. If it is, he increases the thrust to assist in making the correction; if not, he simply continues the landing sequence. This series of tests is continued until a landing is effected, concluding the flight.

N. ROLL AND YAW LOGIC

The computer logic for the study of motion about the roll axis is similar to the logic employed for the study of motion about the pitch axis. The logic of the yaw control study departs somewhat from the pitch and roll logic. The required orientation was specified to be one in which the operator faced the target with an additional test being made, which over-rode the others, to assure that the yaw rate with respect to the ground was not excessive (a maximum allowable rate of 1 radian per second was assumed for this study).

O. MODEL OPERATOR

The physical characteristics of a typical operator were defined for the computer study and appear in Section II, Reference 1. The relationship of physical characteristics, such as locations of the center of gravity and moments of inertia, to the equations formulated for the computer study will be described.

1. Quasi-Rigid Body

The model operator was assumed to be a quasi-rigid body composed of two rigid components hinged together. The components represent the upper and lower parts of the operator's body and the hinge represents the hip joint (Figure 5). The weight of the upper body is 95.2 pounds and the weight of the legs is 64.8 pounds. The distance from the operator's heels to the hip pivot point is 39.5 inches. A SRLD is placed on the operator's back which, it is assumed, weighs 75 pounds empty and is capable of containing 45 pounds of fuel.

2. Centers of Gravity for the Man Machine Combination

The location of the C.G. of the man-machine combination is dependent upon the location of the individual C.G.'s of the operator and SRLD at a given time. The C.G. of the man-machine combination for the initial and burnout condition is shown in Figures 6 & 7. Figure 6 indicates the location of the C.G. for the SRLD and upper body for a full and empty fuel tank. The C.G. of the leg is also shown to be located 17 inches below the hip pivot point, which corresponds to the distance r_2 illustrated in Figure 2, Appendix A.

The location of the combined upper C.G. of the body and SRLD is plotted as a function of fuel consumed in Figure 7. Because of the unique shape of the fuel tank, the location of this composite C.G. will shift laterally to one side of the vertical center line and back again as fuel is consumed. This phenomenon is illustrated in Figure 8.

3. Moments of Inertia for the Man-Machine Combination

Moments of inertia about the pitch, yaw, and roll axes for the composite of the upper body and SRLD in the loaded and empty conditions and the legs were computed. Results are tabulated below.

MOMENT OF INERTIA, SLUG FT^2

	Pitch	Yaw	Roll
Upper body and SRLD Propellant Tank Full	3.30	1.79	3.61
Upper Body and SRLD Propellant Tank Empty	2.74	1.31	3.14
Legs	2.26	0.214	2.40

4. <u>Centers of Pressure and Body Drag</u>

The location of the centers of pressure (C.P.) for the upper body and legs are illustrated in Figure 5. A flat-plate analogy was assumed for computing the locations of C.P. and the drag forces, D_1 and D_2 and D_3 shown in Figure 4, Appendix A, were computed as described in Reference 3. The drag factors, f_1, f_2, and f_3 appearing in the equations on Page 10, Appendix A, were computed to be 0.002, 0.005, and 0.003 respectively.

F. FLIGHT DYNAMICS WITH MOVING CENTER OF GRAVITY

The first series of flights considered were "on design" flights with no perturbations due to winds, body contortions, thrust loss, etc. The method of approach used was to determine a control geometry which allowed stable flight with the operator and system capabilities as defined. Various control geometries were tested, with the intent of producing a flight of 100 foot length at a 30 foot cruise altitude. However, the first group of flights was unsatisfactory, a trajectory typical of this group being that shown in Figure 9. Here, the vehicle flew backwards for almost 500 feet, with ground impact occuring at vertical and horizontal velocities of 27 and 28 feet per second, respectively. The operator, during the first portion of the flight, evidenced an incapability of holding a forward pitch orientation. Later in the flight the problem changed with the operator pitching further forward at each pitch oscillation. Investigation showed that the problem was created by the shift in C.G. as propellant was consumed. As stated earlier, it was considered necessary that the range of motor deflection in pitch bracket the entire range of C.G. travel. A pivot position directly above the C.G. location with the propellant half expended was selected to minimize the motor torque asymmetries. This is obviously only a compromise, with asymmetries still present. The motor was positioned 11 inches above the mean C.G. for the trajectory illustrated in Figure 9. The limits of the motor pitch travel were plus and minus 8°. (Later studies indicate that these are near optimum values). The operator impacted at a pitch angle of 83° for the case under discussion. The small figures super-imposed on the figure show the pitch orientations at the peaks of the pitch oscillations. The pitch angle (ϕ), pitch rate ($\dot{\phi}$), and motor deflection angle (θ_{MOTOR}) during the final three seconds of the flight are plotted versus time in Figure 10. As can be seen by comparing the slopes of the portions of the $\dot{\phi}$ curve corresponding to positive and negative motor deflections, the angular acceleration due to the motor being deflected to + 8° is considerably higher than that due to a deflection of -8°. Since the operator was operating the controls in response to excess pitch rates, with equal reaction time and control force in both directions, there is considerably more overrun of the pitch rate magnitude in the forward pitching direction, causing the mean pitch rate to be forward. This results in a divergence of ϕ with time.

Since the configur tion of the system does not allow the horizontal thrust component to be controlled in a manner completely independent of pitch orientation, control of horizontal position and velocity is lost if the operator is unable to dequately control his pitch orientation. The flight depicted in Figure 9 illustrates the point. During the first part of the flight, the operator was able to retain some degree of control of pitch orientation in that the pitch angle did not diverge. However, he was unable to maintain sufficient control of pitch orientation to allow him to hold the system in the pitched forward altitude required for forward translation, and the large translation backwards resulted.

Q. ALLOWABLE PITCH RATE

The maximum allowable pitch rate was set at 10 degrees per second. It was found that a larger rate (20°/sec was tested) lead to loss of control in the first pitch oscillation. A smaller value (5° second was tested) caused the same type of divergence as was exhibited in Figure 10 and was felt to be less satisfactory than 10°/sec since it required that the time during which a pitch control setting was held be less than the operator's reaction time (and was possibly too small to be observable).

R. SOLUTION OF C.G. SHIFT PROBLEM

There are two possible solutions to the C.G. shift problem, either by designing the system so that the C.G. shift is along a vertical line (this may be accomplished by using a chair-like vehicle with the fuel tank beneath the seat, but the weight increase would make this impractical), or by having the operator trim out the C.G. by moving his legs. The latter alternative was selected as being more practical and the leg motion required was determined. It was found that the legs would have to be bent forward from the hip 12.6° at launch, and gradually moved to the position of bent back 23.3° at the knees at burnout. This is a reasonable range of leg motion which a real operator would be capable of effecting. It would be necessary for the potential operator to learn this C.G. trimming motion before free flight tests were attempted. A synthesis of this trimming was added to the pitch plane program and further trajectories were run. The tesults of these runs are discussed in Section U.

S. THROTTLE LINKAGE

The maximum and minimum thrust levels having been previously chosen, it was necessary to next determine throttle linkage geometry which was compatible with performance and safety requirements, and with sound mechanical design practice. The propellant flow control valves which were investigated exhibited high damping which made rapid operation difficult. For this reason it was decided that the operator should be provided with a high mechanical advantage in the thrust setting range which would be used in flight. A large angular motion may be achieved by the operator if a twist grip is provided. Additionally, if a detent is provided to keep the thrust from dropping below the idle setting, once the throttle has been opened this far, the operator may reposition his hand after bringing the thrust up to idle. It was determined that the throttle valve would have to be opened 75° to bring the thrust up to the idle value, with the other 15° of rotation bringing the thrust up to the maximum value. A practical cam actuated control system could be built which effected the first 75° of throttle opening with 50° of control rotation, and the final 15° of throttle rotation with an additional 50° of control rotation. This would provide an average mechanical advantage in the flight operation range of thrust setting of 3.33:1. The cam which was designed had thrust variation with control setting in the flight range as depicted in Figure 11. Starting with this contour, different spring and damping rates for the control lever were studied. It was determined that the operator would be capable of providing approximately 3 foot pounds of torque to the control (Reference 1) and the moment of inertia of the rotating components was estimated to be .05 slug-feet squared. It was found through study of a series of computed altitude approach trajectories with varied control parameters that the operators altitude holding capability improved as the damping and spring rates were reduced. High damping resulted in slow throttle control with attendant large overshoots of the desired altitude. The damping was eventually completely removed. It was determined that when a strong spring was included in the system to close the throttle, the capability of the operator in approaching the desired altitude from below and stopping at the desired altitude was improved, but that his approach performance in descent deteriorated. It was finally determined that the only spring that should be provided was a light one to return the control to the idle setting when the control was released. This is considered desirable from a safety standpoint, since if the operator should find cause to temporarily release the controls during flight, the minimum of attitude perturbation due to thrust moments would result at idle (it is considered undesirable to allow the motor to shut down during flight, though this of course, would result in zero thrust torque). The control motion for the system selected, is defined by the equation:

$$M - K_s \Theta = J \frac{d^2\Theta}{dt^2}$$

where M is the applied moment (plus or minus 3 foot pounds or zero), K_s is the spring coefficient (.15 foot pounds per radian), Θ is the control deflection in radians (.874 minimum, 1.748 maximum), J is the polar moment of inertia (.05 slug feet squared), and t is time in seconds. The contour of the cam designed provided the thrust variation shown in Figure 11. The equation of this curve is

$$\text{Thrust (pounds)} = -37.55\,\Theta^2 + 273.3\,\Theta - 52.6$$

(Θ in radians)

It may be expected that the best performance in altitude holding will occur when the available upward and downward acceleration rates are equal. The most critical altitude-velocity control problem exists at landing. Since it may be expected that flights would be conducted for near total propellant duration, the earlier thrust level choices are fortuitous since the system weight at burnout would be halfway between the idle and full thrust levels thus making the upward and downward accelerations equal.

It was determined through analysis of trajectory data for flights at several design altitudes and ranges that after pitch control was refined to allow good attitude control, the thrust control, as described, allowed consistantly safe flights and landings with a cruise altitude of 15 feet being specified. It was determined from computed trajectories with various values of h_c that the decision to pitch forward and translate horizontally could be safely made when an altitude as low as 2 feet was reached.

T. LANDING RADIUS

It became obvious during the course of the computer runs that accuracy in braking at a desired horizontal displacement was quite poor, and the originally specified Δx_ℓ (allowable landing radius) of 10 feet was much too stringent a requirement. In the trajectories in which this value was specified, the vehicle ran out of fuel while the operator was attempting to correct the overshoot which always occurred. To allow safe landings, so that complete flights could be studied, Δx_ℓ was arbitrarily increased to 300 feet. The landing logic diagram (Figure 4) indicates a test of the suitability of the operator's horizontal position for landing (is $|x_R - x| > \Delta x_\ell + K_4 h$?). The Δx_ℓ of 300 feet prevented the answer to this test from being "yes" for all the cases which were computed, so that the command to increase thrust so as to forestall landing in the event of a "yes" answer was always bypassed.

U. PITCH CONTROL GEOMETRY

Figure 12 shows the thrust vector deflection in pitch which is required to bracket the C.G. travel as a function of the elevation of the motor pivot axis above the hip pivot axis. Since the type of gimbal considered could not be deflected more than 8°, the motor pivot axis would have to be at least 8 1/2 inches above the hip pivot. A position 17.16 inches above the hip pivot (11 inches above the mean C.G.) was found to be desirable from a hardware geometry standpoint, and this position was selected as a basis for study. It was assumed that the motor deflection was equal to the control deflection. A minimum pitch deflection of slightly less than 2° would be required; however, since operators will have varying builds, a somewhat greater deflection should be provided. A range of deflection limits of 4° to 8° was studied. Supplementary studies performed earlier on the IBM 610 digital computer indicated that the use of a return spring, which would act as a booster, would enhance the pitch control stability.

The motion of the control, in returning to the central position is computed assuming the spring force and the maximum force output of the operator both to be acting on the control. Since the operator's muscles have a limitating rate of deflection, regardless of load, this is not always the case. If the control deflection rate due to the spring alone is extremely high, the operator would not be capable of exerting maximum force in the direction of motion. It is felt however, that in the control configurations considered herein, the deflection rates are not this high. As the operator discovers that the control deflection which he has applied has resulted in a condition of overcorrection, he makes a decision to reverse the moment on the control. During the period of his reaction time plus the time required to return the control to the neutral position, the angular acceleration due to his original control deflection continues, increasing the pitch rate in the direction of the overcorrection. Adding the spring reduces the time during which the control deflection is in the wrong direction. This results in an improvement in pitch control. Figure 13 shows the variation of deflection with time for a control of this type. It was assumed, in computing this curve, that the operator applied a positive 10 foot pound moment until the motor reached the deflection limit which brought it to rest (at .3 seconds) and then immediately applied a negative moment of 10 foot pounds until the other stop was reached. The control geometry illustrated in this figure was found to have too slow a response due to excessive damping, and pitch attitude errors of up to 50° were noted. Damping rates were reduced until satisfactory pitch control was obtained and these values should be considered allowable maximums. The spring rates were adjusted so that the restoring moment due to the spring at full deflection was just slightly less than the moment the operator could apply. Figure 14 illustrates attempted 50 foot flights at a 15 foot altitude with different maximum motor deflections, and spring and damping rates. For the case in Figure 14a the spring rate was 143.0 foot pounds per radian, the damping rate was 7.50 foot pound seconds per radian, and the maximum deflection was 4°. In Figure 14b, the corresponding values are 95.0, 5.0 and 6° and in Figure 14c they are 71.0, 3.75, and 8°. The magnitude of the pitch oscillations was approximately the same in all three cases, and there was no apparent advantage to any of the configurations over the others on this basis. In the case in Figure 14a, the propellant ran out before a landing could be effected, and landing occurred at a forward velocity of 32.9 feet per second, a downward velocity of 23.9 feet per second, and a pitch orientation of -32.2°. In the case in Figure 14b landing occurred at a forward velocity of 21.5 feet per second, a downward velocity of 2 feet per second, and a pitch orientation of $+9.0^{\circ}$, 37.8 pounds of propellant were expended. In the case in Figure 14c, landing occurred at a forward velocity of .1 foot per second, a downward velocity of 8 feet per second, and a pitch orientation of -2.5°. The pitch rate at touchdown was $+24.4^{\circ}$ per second. This was the only case which completely satisfied the landing requirements and was the configuration selected for further study. The effect of variation of the pivot position was investigated, the trajectories being shown in Figures 14c [illegible]ugh 17. The control geometry, other than pivot location is that selecte[illegible]e. Within a range of pivot heights from 14.4 to 17.16 inches above the hip pivot, there is little difference in system performance. With a motor pivot height of 12 inches (Figure 17) a landing occurred at 67.5 feet which is quite close to the desired landing point. It is thought that this was rather accidental since the position control exhibited at launch is poor. Landings were performed within the velocity and pitch orientation limits prescribed except for the case of a motor pivot 19.2 inches above the hip pivot, in which case the system goes out of control positionwise, though attitude was being held well for the portion of the flight which was computed (Figure 16). It was decided that the thrust pivot elevation of 17.16 inches (11 inches above mean C.G.) would be the one consider d for the balance of the flights. This is the position used for the flight illustrated in Figure 14c. A plot of the pitch rate and orientation for several seconds of this flight is presented as Figure 18 for comparison with Figure 10 to show the effect of C.G. trimming through leg motion. Note that the pitch angle does not diverge in Figure 31 thus indicating a stable condition.

A vertical jump was the next maneuver to be studied. The operator was instructed to accelerate upward to 30 feet and then brake and descend. This maneuver was performed in a safe and stable manner, though with a horizontal displacement of 64.4 feet from the intended landing spot. Touchdown was accomplished with a downward velocity of 5.7 feet per second, a forward velocity of 4.6 feet per second, a pitch orientation of -16.1° and a pitch rate of +7.9 degrees per second. There were 43.6 pounds of fuel expended of the 45 pounds available, and the peak altitude of 33.2 feet can be consider d the maximum vertical performance to be expected of this vehicle with an operator possessing the capabilities of the one considered in this study. This flight is illustrated in Figure 19.

V. ABNORMAL FLIGHT CONDITIONS

Since the control moments available to the operator and the upsetting moments which might cause severe aberrations of the flight plan are internal, (the center of pressure is quite close to the C.G. and the aerodynamic moments on the system will be quite small) attitude control is independent of spatial orientation, and all that would be required to regain control of the vehicle, after the abberation is removed, is sufficient time. The abnormal flight conditions that may adversely affect the accomplishment of flight objectives are: (1) loss of attitude control or judgment, (2) temporary loss of thrust, (3) unusual contortion of the body in flight.

The effect of each of these perturbations is to temporarily cause the loss of the vertical thrust component. Since the thrust to weight ratio of the vehicle being considered is quite low, any downward velocity which accumul tes during a temporary thrust loss will take measurable time to correct. Obviously, this may result in the loss of considerable altitude. If it is assumed that the thrust loss occurs at a vertical velocity of zero, that the initial weight is equal to the takeoff weight, and that the maximum safe impact velocity is 20 fps, then the minimum altitude at which the vertical thrust component could normally be lost without injury to the operator varies with the duration of the thrust loss as follows:

Initial Altitude	Allowable Duration of Thrust Loss
25 feet	.97 seconds
50	1.04
75*	1.10
150	1.30
300	1.70
865	3.00

* With duration and thrust specified for the model SRLD and assuming 100% total impulse contributes to vertical flight, the maximum altitude attainable is 80 feet for a takeoff from and safe return to ground level.

The effect on the operator's attitude of a violetn body contortion was determined by computing a trajectory containing a 90° forward kick and return (Figure 20), as detailed in Appendix A. Figure 21 illustrates the variation of the operator's attitude. Body configuration is superimposed on this Figure. It may be seen th t the system has been stabilized after going through only slightly more than 360° of rotation. This should be considered a satisfactory display of regaining control, considering the magnitude of the attitude perturbation during the kick. The obvious problem is the large altitude loss (136 feet) and the accumulation of a downward velocity of over 70 feet per second. Since the altitude loss alone is greater than the altitude attainable with the system, it follows that an attitude perturbation of this magnitude could not be tolerated. On the other hand, the motion of the operators body does show the large degree of kinesthetic control available with a vehicle which allows the operator considerable freedom of motion.

W. YAW CONTROL

Yaw control was investigated by testing the ability of the system to approach a target point in a cross wind. Though this situation is not strictly correct, in that it is assumed that the operator will be pitched forward throughout the flight, it is still a good test of yaw control. The logic which was assumed was that if it appeared to the operator that his bearing was correct (a threshold level of $\pm 5^\circ$ was assumed) he would provide a control moment to oppose any sensible yawing rate relative to the target (a threshold level of $\pm 10^\circ$ per second was assumed). If the operator sensed a bearing error, he would provide a control moment to correct it by heading directly for the target and not overcorrecting to compensate for drift. An over-riding requirement was that the yawing rate relative to the ground be kept below 1 radian per second. The control geometry which was tested was defined by:

$$M - K_s \, \theta_{\text{YAW CONTROL}} - K_D \frac{d\,\theta_{YC}}{dt} = J \frac{d^2\,\theta_{YC}}{dt^2}$$

where M (the moment applied by the operator) was plus or minus 2 foot pounds or zero, the spring rate (K_s) was .5 foot pounds per radian, the damping coefficient (K_D) was .5 foot pound seconds per radian, the polar moment of inertia of the control plus the operator' arm was .5 slug ft^2 and θ_{YC} was the control deflection in radians. The control was assumed movable between +11.5° and -11.5°. The jetavator was assumed to be deflected .313 degrees per degree of control deflection. This gives a maximum jetavator deflection of 3.6° and yeilds a resultant 2.5 foot pounds of torque at full thrust. An attempted flight to a target 30 feet distant with a 30 foot per second cross wind produced the trajectory depicted in Figure 22. As the operator nears the target he finds that the yaw rate required to hold his heading towards the target becomes excessive. However, while still at some distance from the target he is capable of holding his heading within a few degrees of the perfect orientation. This is shown by the vectors superimposed on the curve which show the operator's heading. With the geometry chosen, the operator was able to keep control of the vehicle, and to pass within 6 feet of the target as opposed to a 15 feet miss distance if no attempt is made to vary the yaw orientation. The geometry chosen was considered satisfactory for safe flight.

X. ROLL CONTROL

A roll control geometry was selected and tested, two cases being shown in Figure 23. The operator logic involved was the same as that used in the pitch control studies. The two cases shown differ in that the damping rate in Case 1 is twice that of Case 2. The maneuver involved was to move from a roll orientation of 10 degrees to vertical position. The performance of the system in Case 2 is considered satisfactory. The control system is defined by:

$$4 - 4\theta_{\text{ROLL CONTROL}} - 1 \frac{d\,\theta_{RC}}{dt} = 0.5 \frac{d^2\,\theta_{RC}}{dt^2}$$

where the terms are equivalent to the corresponding terms defined in the description of the yaw control geometry, (θ_{RC} is in radians). This control is assumed to operate a valve which varies the thrust of one motor to provide a rolling moment. The control is movable between +30° and -30°. The linkage between the control and the valve is such that the differential thrust is equal to $\pm 40\,\theta^2_{RC}$ where θ_{RC} is in radians. This gives a maximum differential thrust of 10 pounds at full thrust.

V. SIGNIFICANCE OF THE SYSTEM DEVELOPED

It should be pointed out that the system studied is one answer to the problem of providing a small rocket lift device which is capable of acceptable stability and performance. The controls which were defined are by no means the only set which would allow acceptable flights. The optimisation of the system is beyond the scope of this study, which is aimed at demonstrating one approach to solving the control problems associated with a SRLD. It is felt that the configuration determined herein is one which satisfies the multiple requirements of safety, performance, simplicity, light weight and compactness.

SYMBOLS

		UNITS
C_o, C_1, C_2	Gain for Control Linkage	-
C.G.	Center of Gravity	-
C.G.	C. G. Shift	ft
D_1, D_2, D_3	Drag Forces	lb
F	Thrust	lb
°F	Temperature, Degrees Fahrenheit	-
H_{R_o}	Initial Required Altitude	ft
I	Moment of Inertia	slug-ft^2
J	Polar Moment of Inertia of Control Linkage and Operator's Arm	slug-ft^2
K_D	Yaw Linkage damping Coefficient	ft-lb-sec/rad
K_1	Spring Constant	ft-lb/rad
K_2	Damping Constant	ft-lb-sec/rad
K_4	Judgement Tolerance Coefficient	-
K_s	Spring Coefficient	ft/sec/ft
M	Applied Moment	ft/lb
RT	Reaction Time	sec
T	Time	sec
V	Landing Velocity	ft/sec
V_x	Velocity in X direction	ft/sec
V_{Y_D}	Downward Velocity	ft/sec
X	Horizontal Coordinate Axis	-
X_R	Required Horizontal Position	ft
X_{R_o}	Initial Required Displacement	ft

		UNITS
X_{R_1}	Range of Target from Takeoff Point	ft
a	Body Component Depth	inches
b	Body Component Width	inches
c	Body Component Height	inches
d	Distance From Body C.G. to Component C.G.	inches
f_1, f_2, f_3	Drag Factors	-
g	Gravitational Constant, 32.2	ft/sec^2
h	Altitude	ft
h_o	Arbitrary Altitude Above the Starting point	ft
h_R	Required Altitude	ft
m	Body Component Mass	slugs
$\dot{m}$	Mass Flow Rate	slug/sec
r	Body Component Radius	inches
x	Distance from Landing Point	ft
α	Angular Acceleration	rad/sec^2
θ	Control Deflection Angle	rad
θ_{motor}	Motor Deflection Angle	rad
θ_{rc}	Roll Control Deflection	rad
θ_{YC}	Yaw Control Deflection	rad
ϕ	Control Lever Displacement	rad
ϕ_{ℓ}	Pitch Angle at Landing	rad
ϕ_R	Required Pitch Angle	rad
Ω	Position Error	rad
$\dot{w}$	Weight Flow Rate	lb/sec

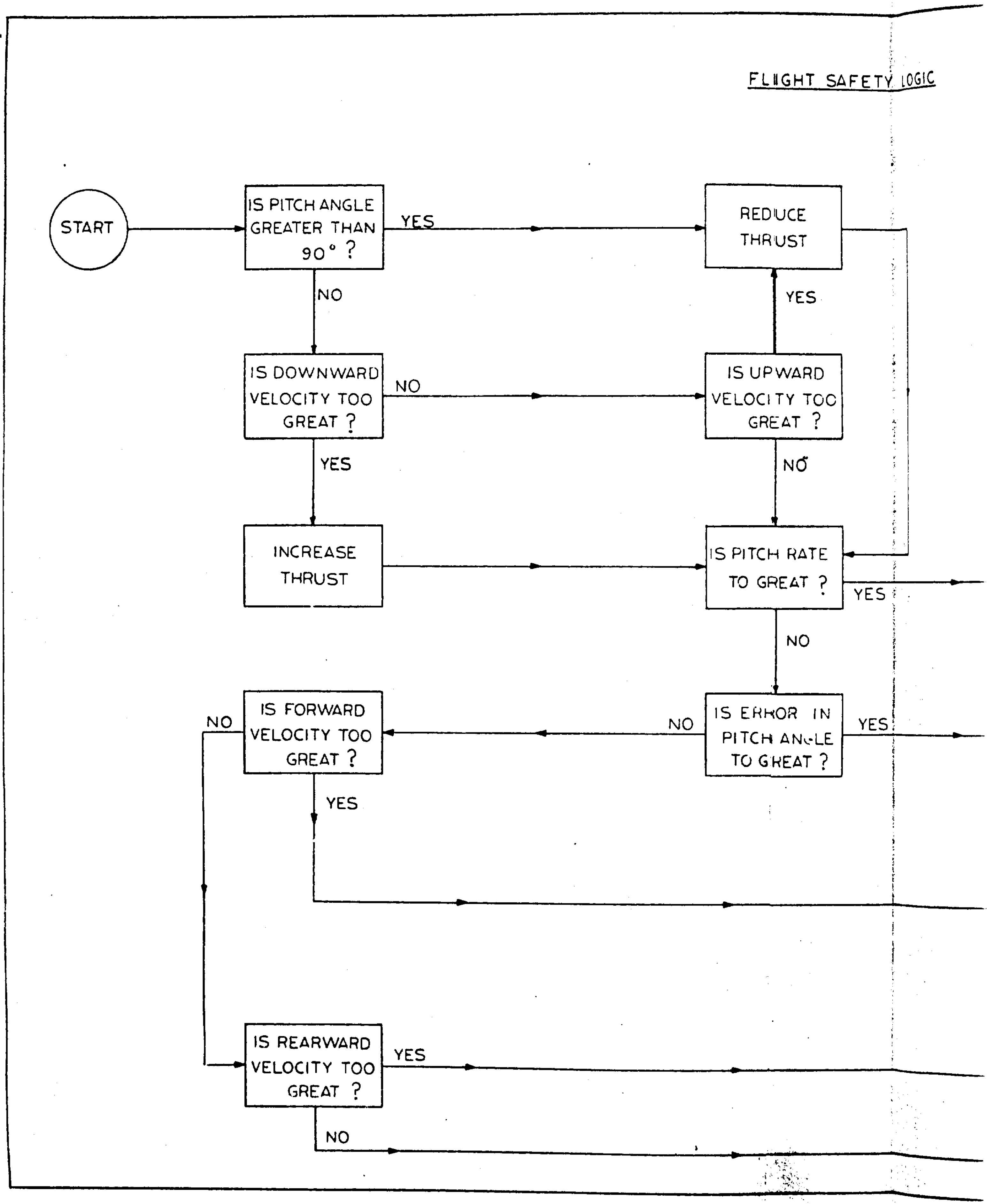
REFERENCES

1. Feasibility Study of Small Rocket Lift Device, Aerojet Report 1751, February 1960

2. Dynamic Response of Human Operator, WADC TR 56-524 ASTIA Doc. No. AD-110693, October 1957

3. Sighard F. Hoerner, Fluid Dynamic Drag, published by author, S. F. Hoerner, 148 Busterd Dr., Midland Park, N. J. 1958

FLIGHT SAFETY LOGIC



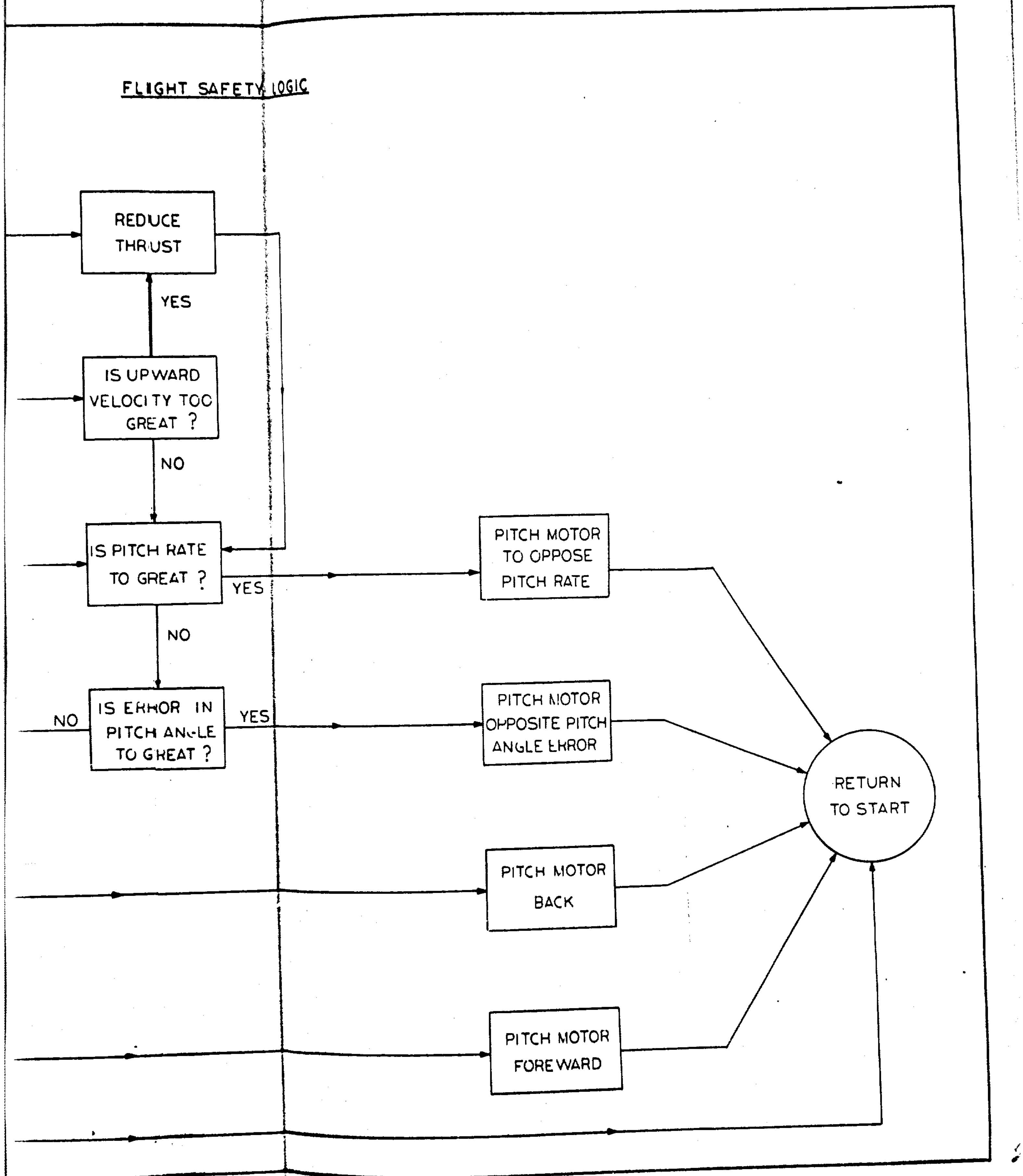

FLIGHT SAFETY LOGIC
REDUCE THRUST
YES
IS UPWARD VELOCITY TOO GREAT ?
NO
IS PITCH RATE TO GREAT ?
YES
PITCH MOTOR TO OPPOSE PITCH RATE
NO
NO
IS ERROR IN PITCH ANGLE TO GREAT ?
YES
PITCH MOTOR OPPOSITE PITCH ANGLE ERROR
RETURN TO START
PITCH MOTOR BACK
PITCH MOTOR FOREWARD

Figure 2

FLIGHT ENVELOPE DETERMINANTS

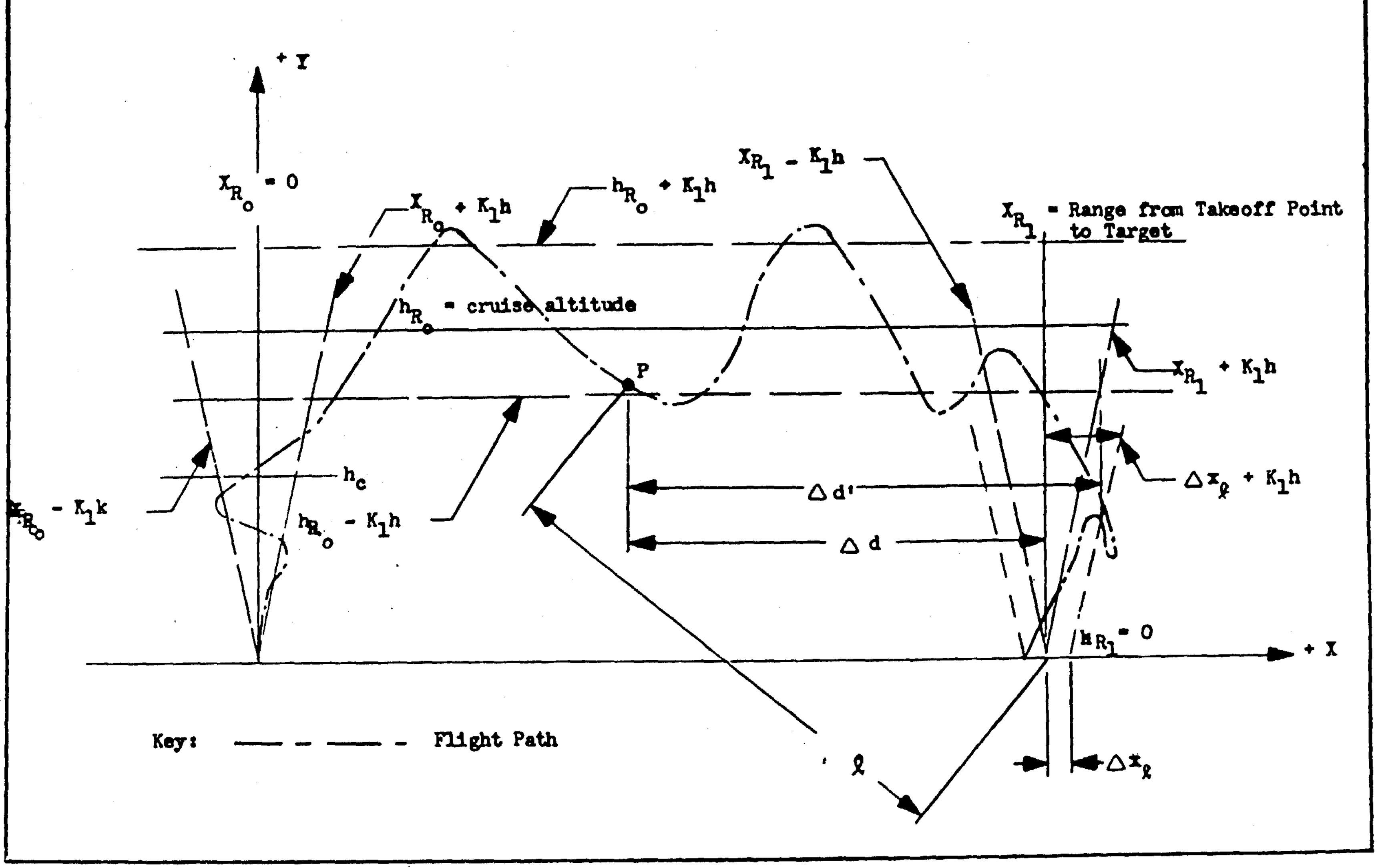


Figure 3

LANDING LOGIC

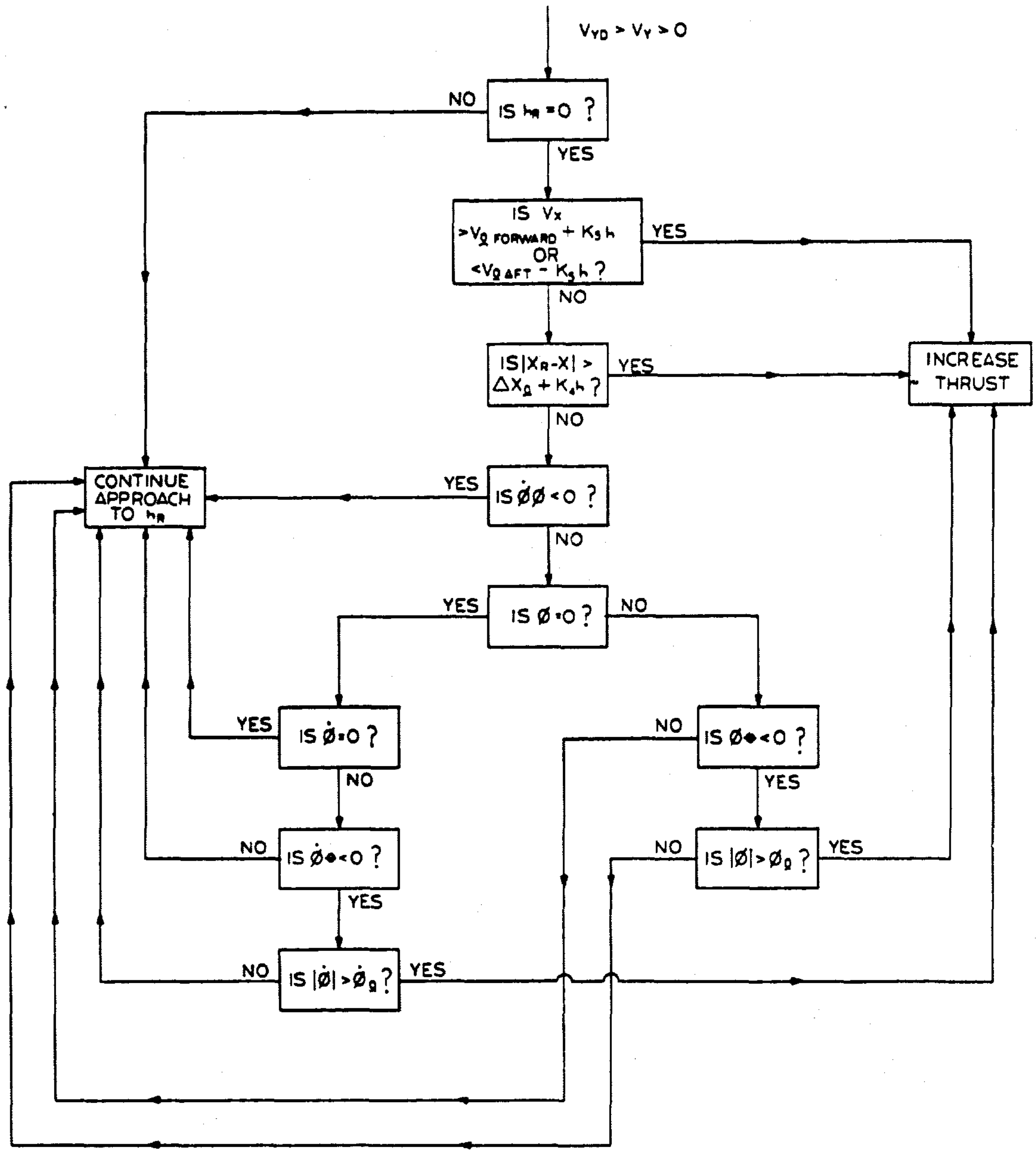


Figure 4

FLIGHT ENVELOPE DETERMINANTS

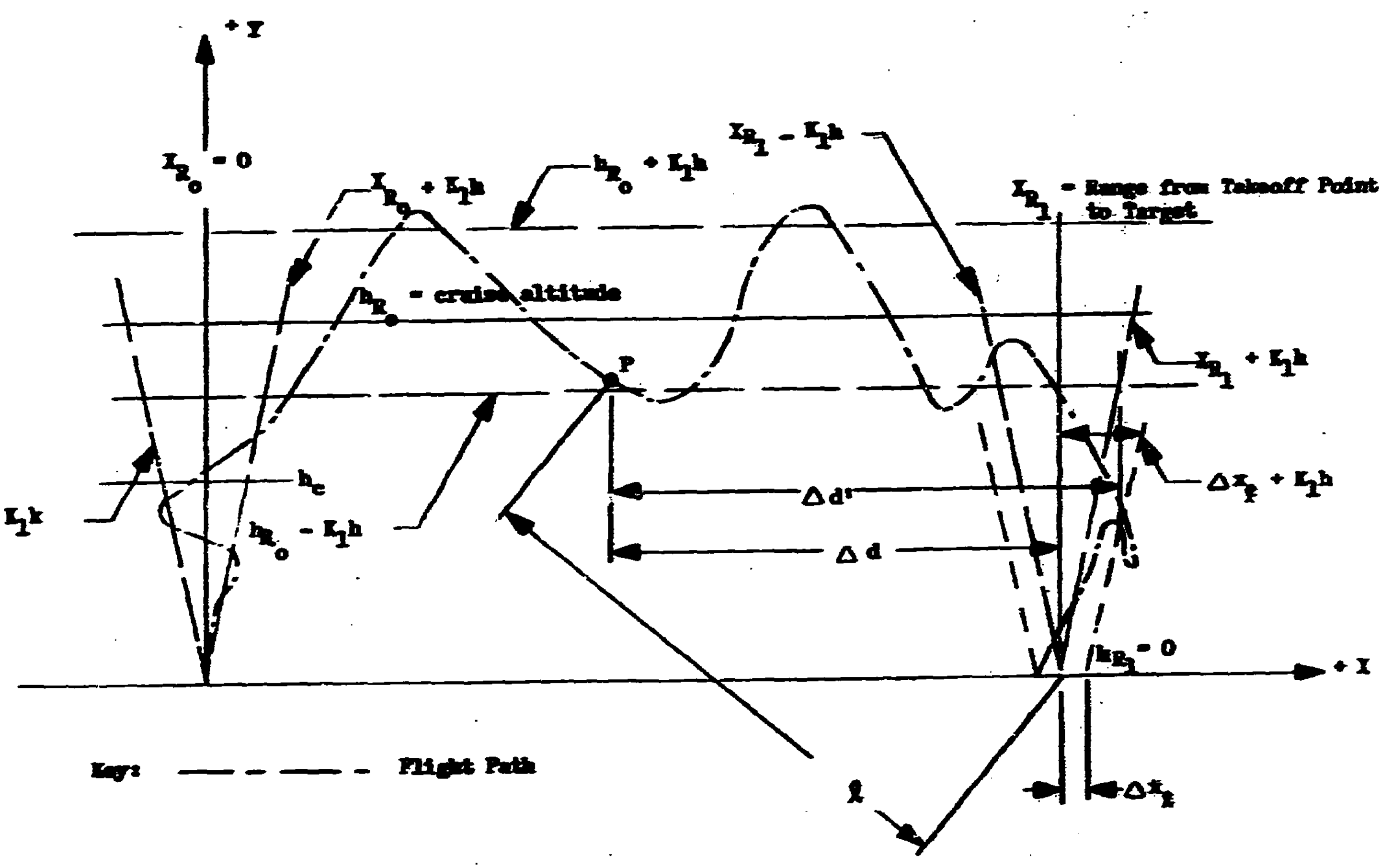


Figure 3

LANDING LOGIC

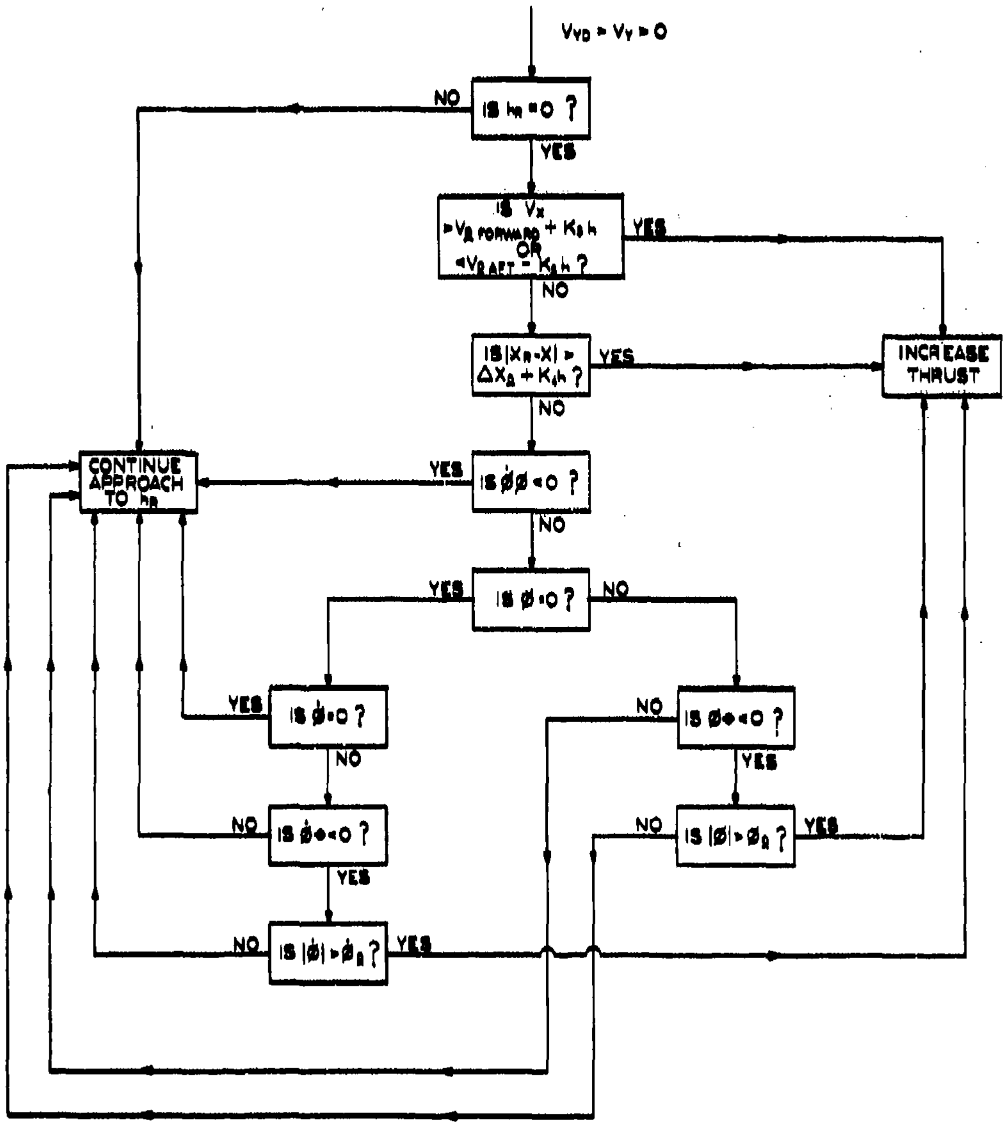
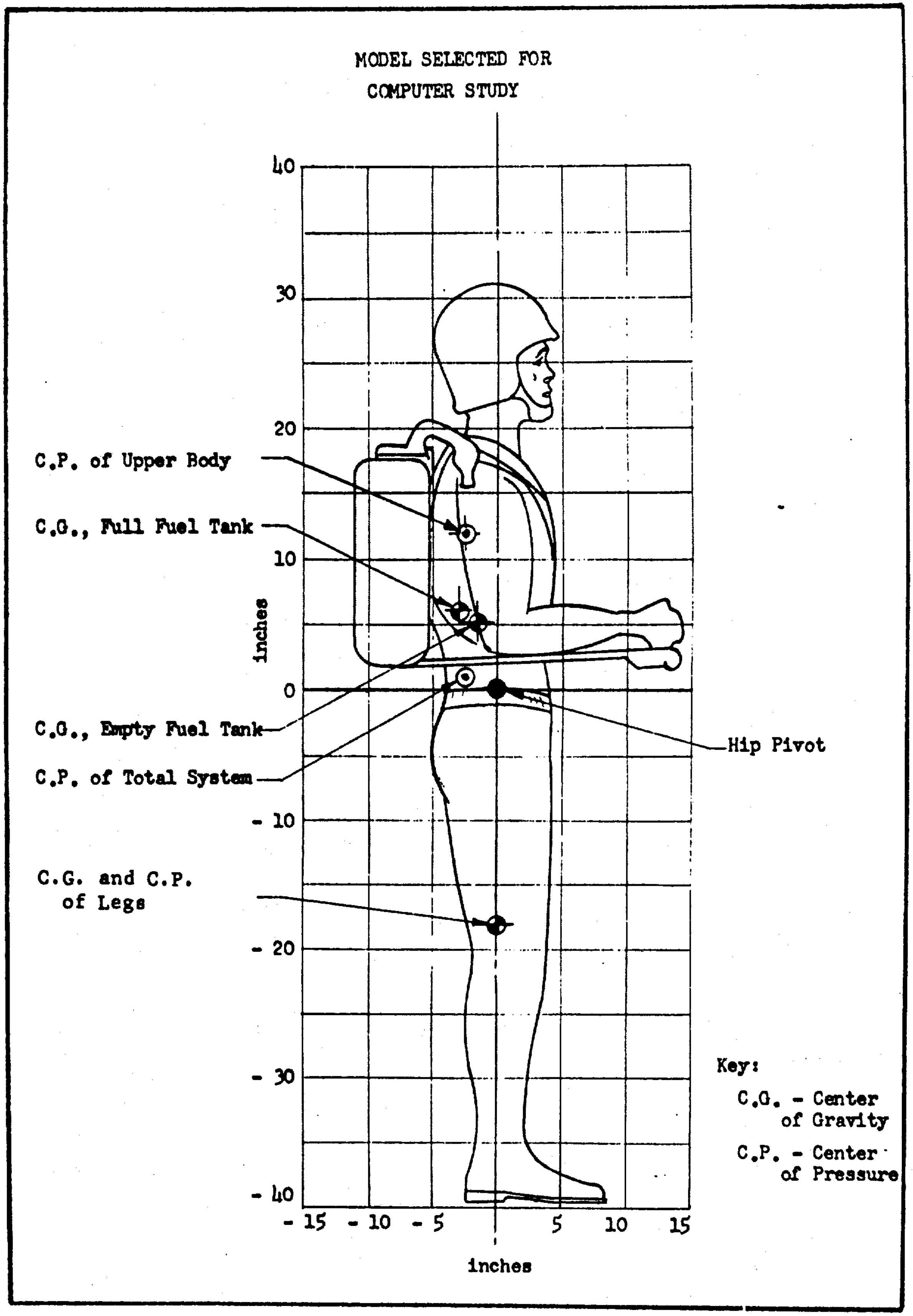


Figure 4



MODEL SELECTED FOR
COMPUTER STUDY
40
30
20
10
0
- 10
- 20
- 30
- 40
inches
- 15
- 10
- 5
5
10
15
inches
C.P. of Upper Body
C.G., Full Fuel Tank
C.G., Empty Fuel Tank
C.P. of Total System
C.G. and C.P.
of Legs
Hip Pivot
Key:
C.G. - Center
of Gravity
C.P. - Center
of Pressure

Figure 5

CENTER OF GRAVITY LOCATIONS

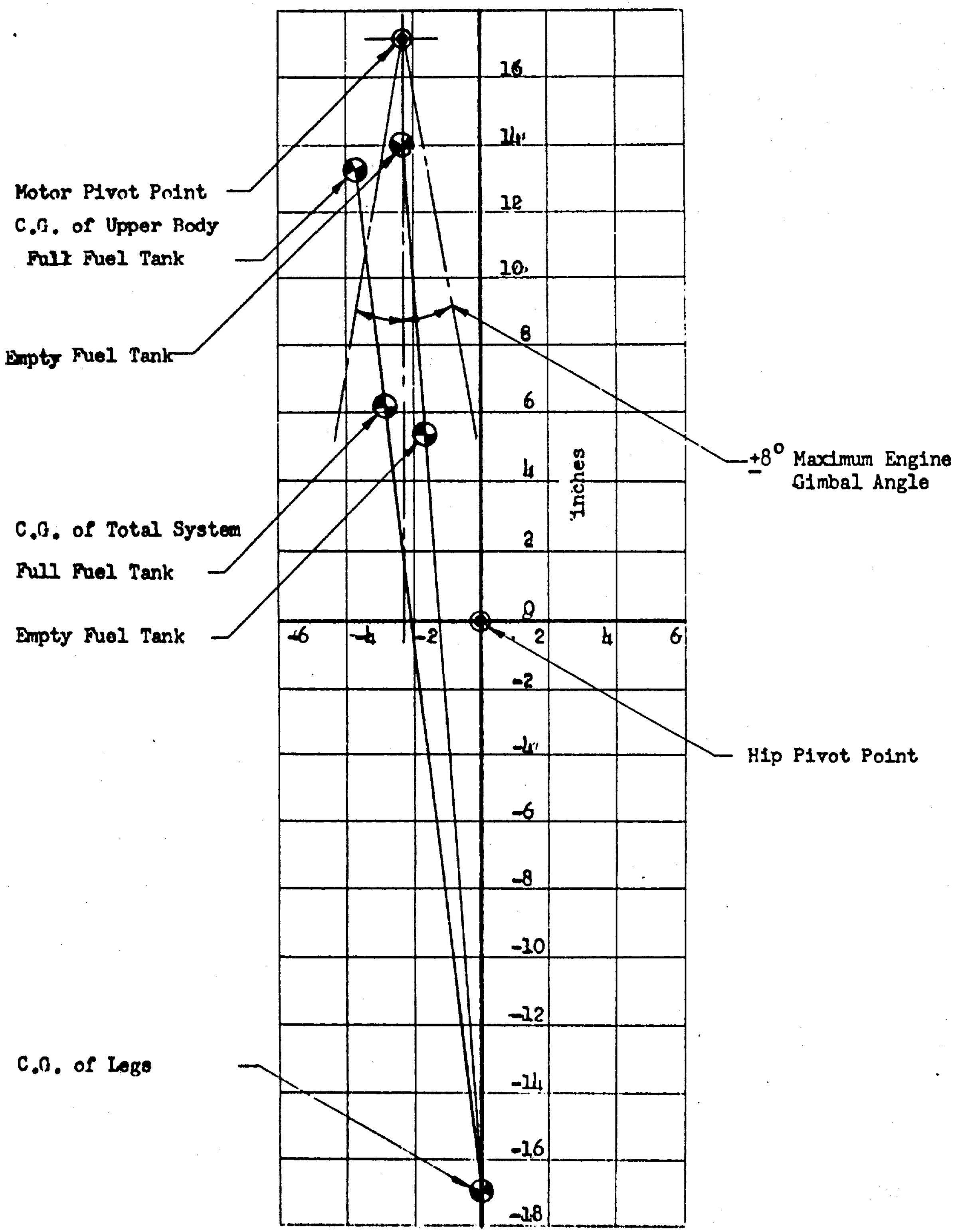
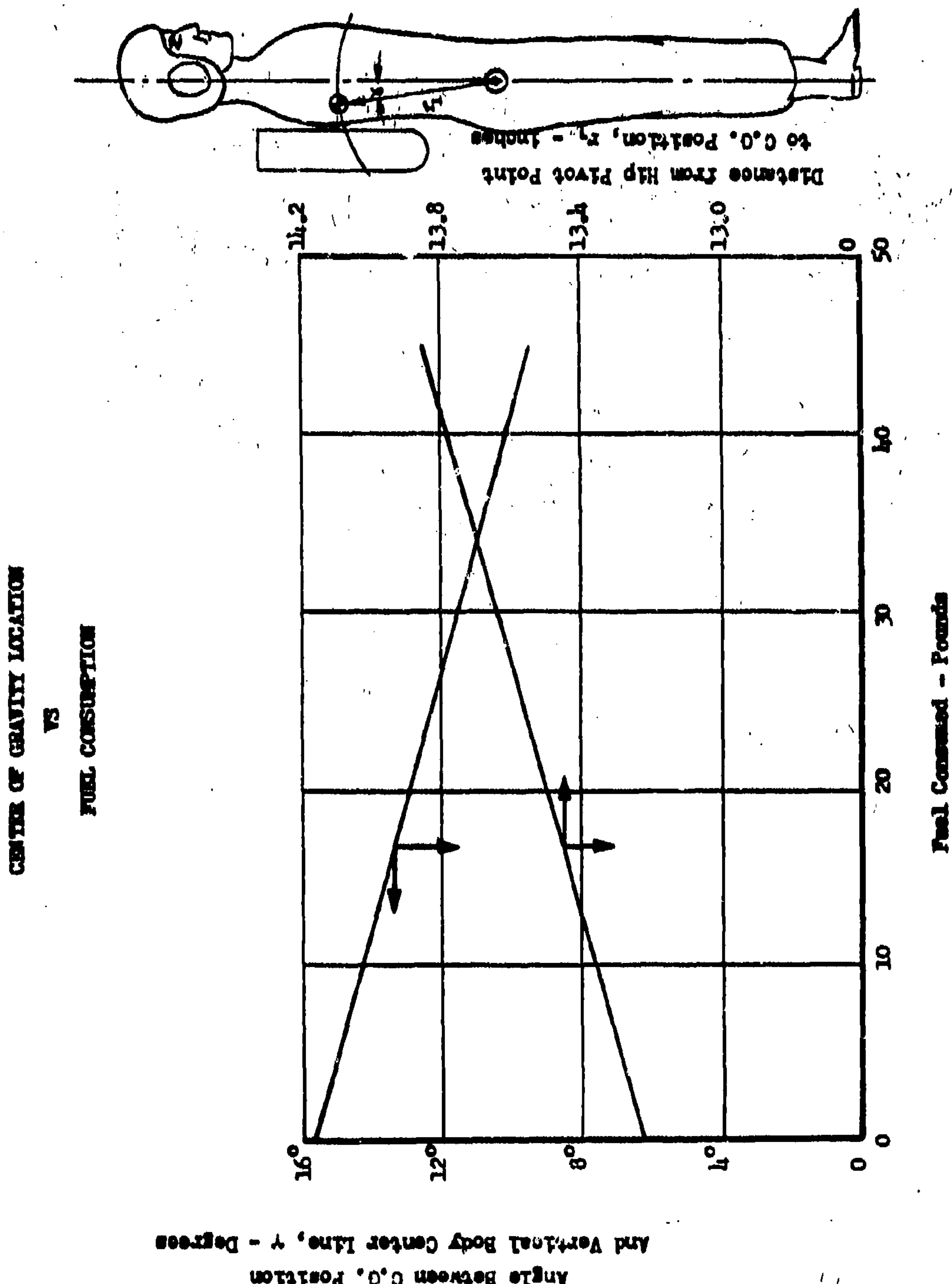


Figure 6



CENTER OF GRAVITY LOCATION
VS
FUEL CONSUMPTION
Distance from Hip Pivot Point
to C.G. Position, r_1 - inches
14.2
13.8
13.4
13.0
0
r_1
γ
16°
12°
8°
4°
0
Angle Between C.G. Position
And Vertical Body Center Line, γ - Degrees
0
10
20
30
40
50
Fuel Consumed - Pounds

Figure 7

LATERAL CENTER OF GRAVITY SHIFT

VS

FUEL CONSUMPTION

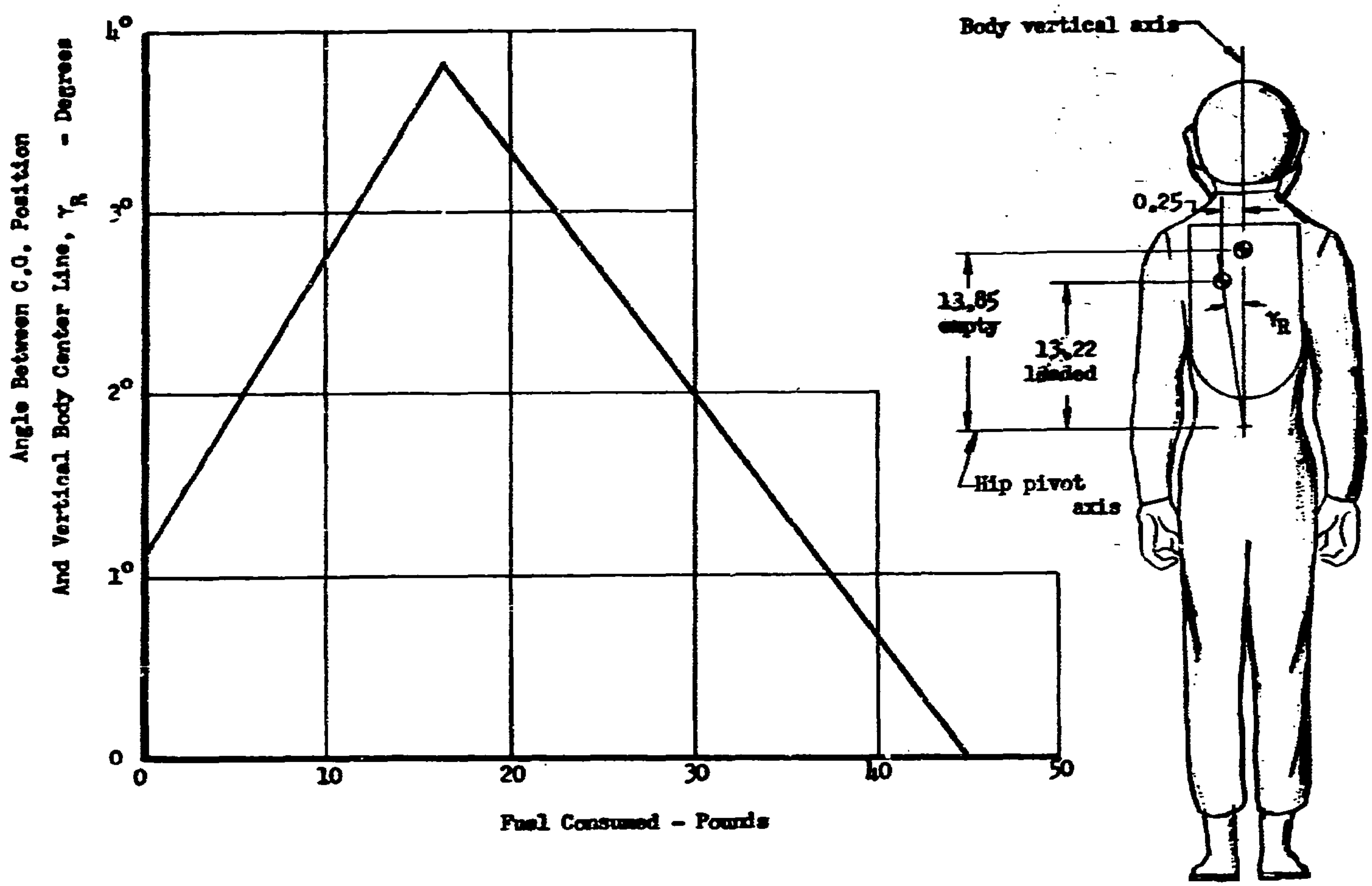


Figure 8

FLIGHT PATH WITH NO CENTER OF GRAVITY TRIM

TRAJECTORY COMPUTED ON IBM 704

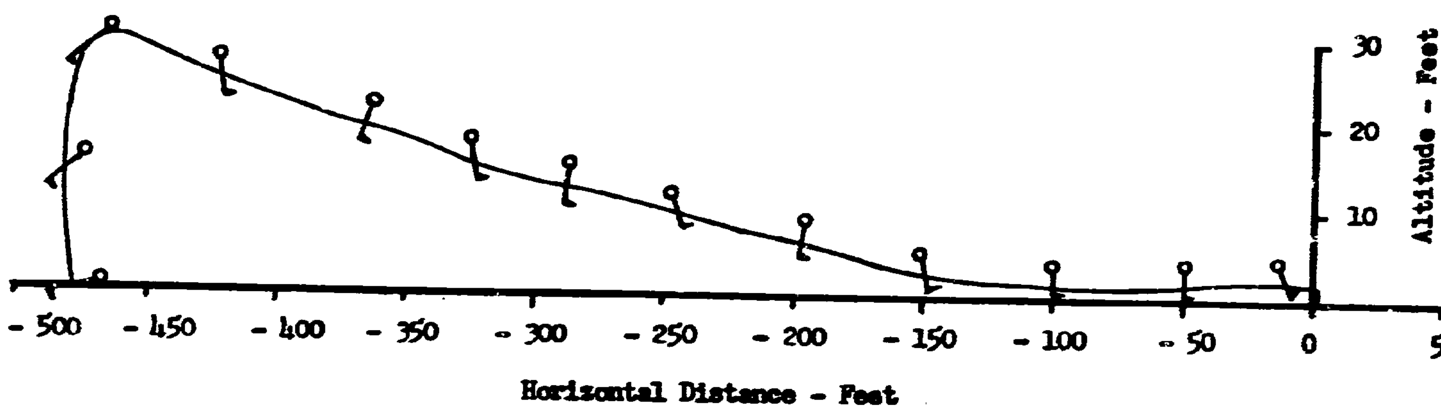


Figure 9

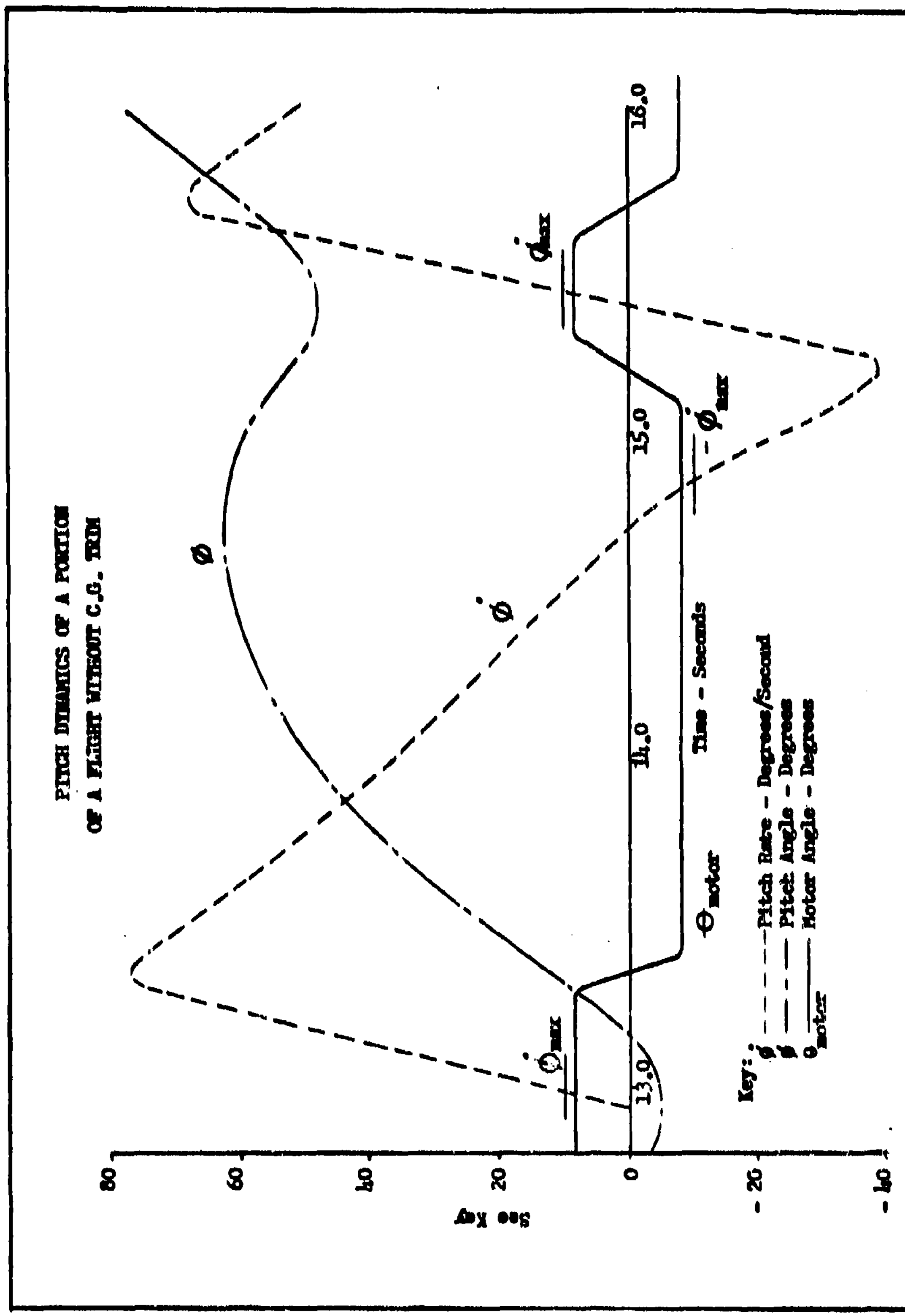

PITCH DYNAMICS OF A PORTION
OF A FLIGHT WITHOUT C.G. TRIM
See Key
Time - Seconds
13.0
14.0
15.0
16.0
80
60
40
20
0
- 20
- 40
Key:
$\dot{\phi}$ - - - Pitch Rate - Degrees/Second
ϕ — - — Pitch Angle - Degrees
θ_{motor} —— Motor Angle - Degrees

Figure 10

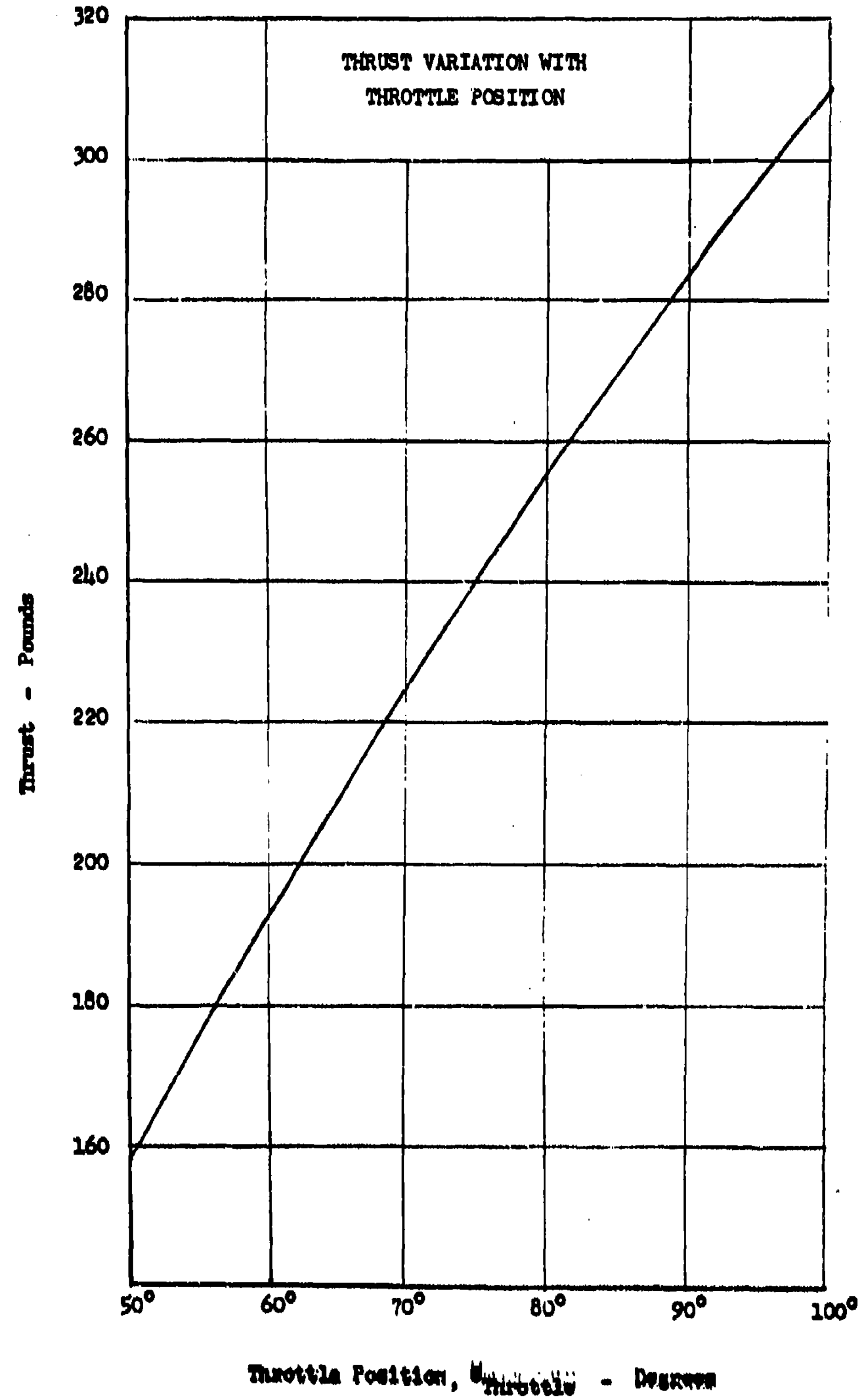

THRUST VARIATION WITH
THROTTLE POSITION
320
300
280
260
240
220
200
180
160
Thrust - Pounds
50°
60°
70°
80°
90°
100°
Throttle Position, θ Throttle - Degrees

Figure 11

MINIMUM MOTOR DEFLECTION ANGLE
VS
MOTOR PIVOT LOCATION

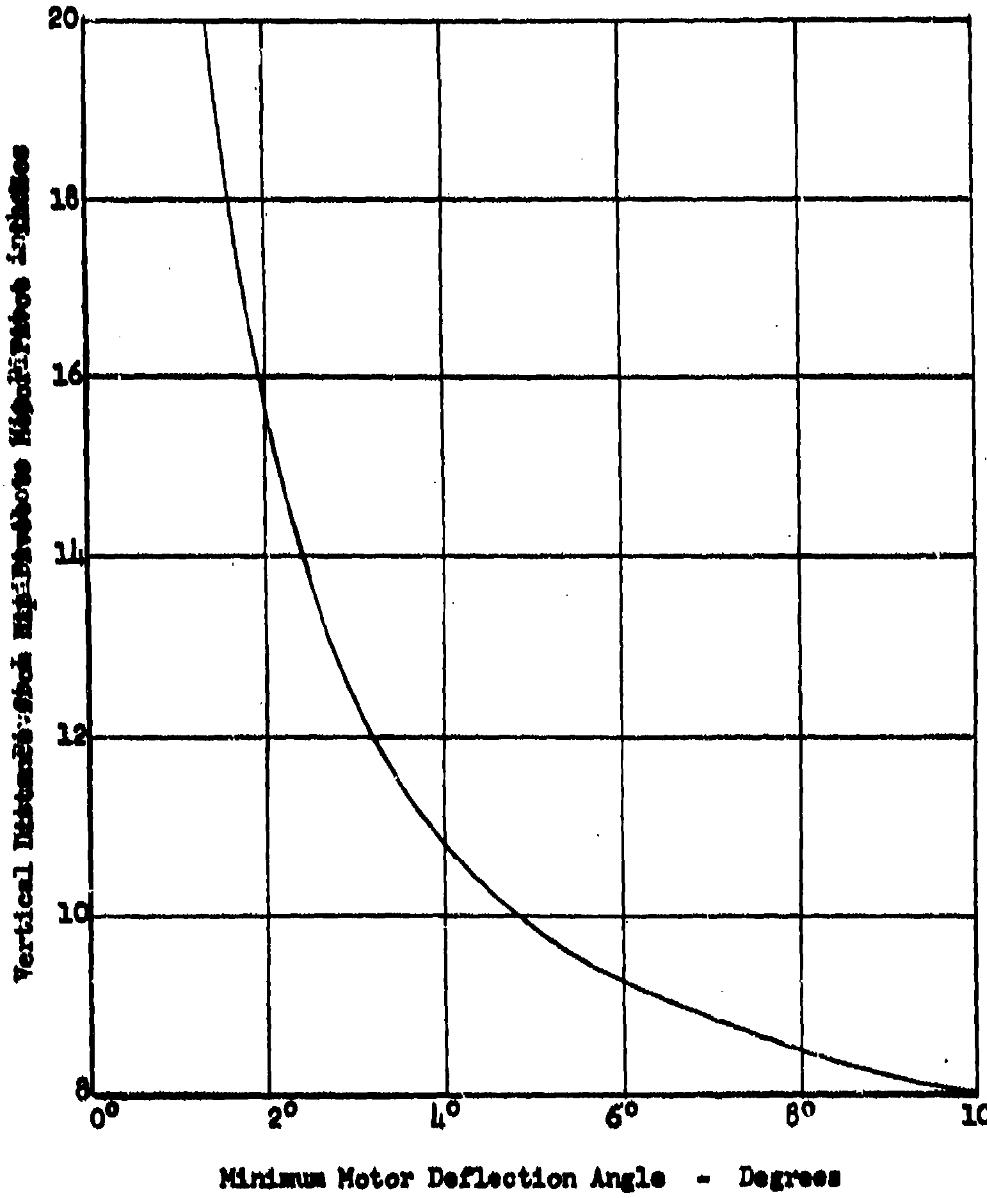


Figure 12

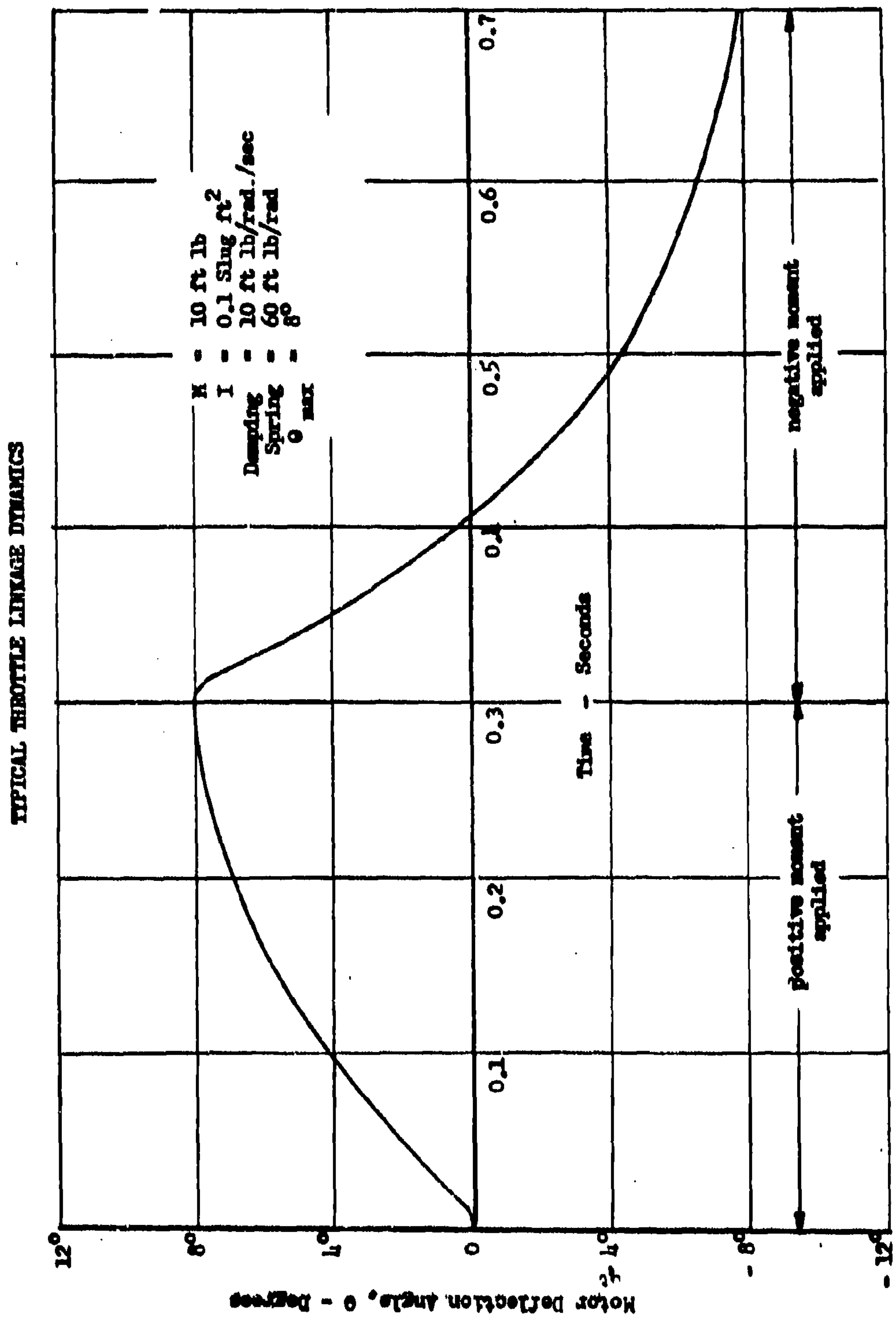

TYPICAL THROTTLE LINKAGE DYNAMICS
M = 10 ft lb
I = 0.1 Slug ft²
Damping = 10 ft lb/rad./sec
Spring = 60 ft lb/rad
θ max = 8°
Motor Deflection Angle, θ - Degrees
12°
8°
4°
0
-4°
- 8°
- 12°
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Time - Seconds
positive moment applied
negative moment applied

Figure 13

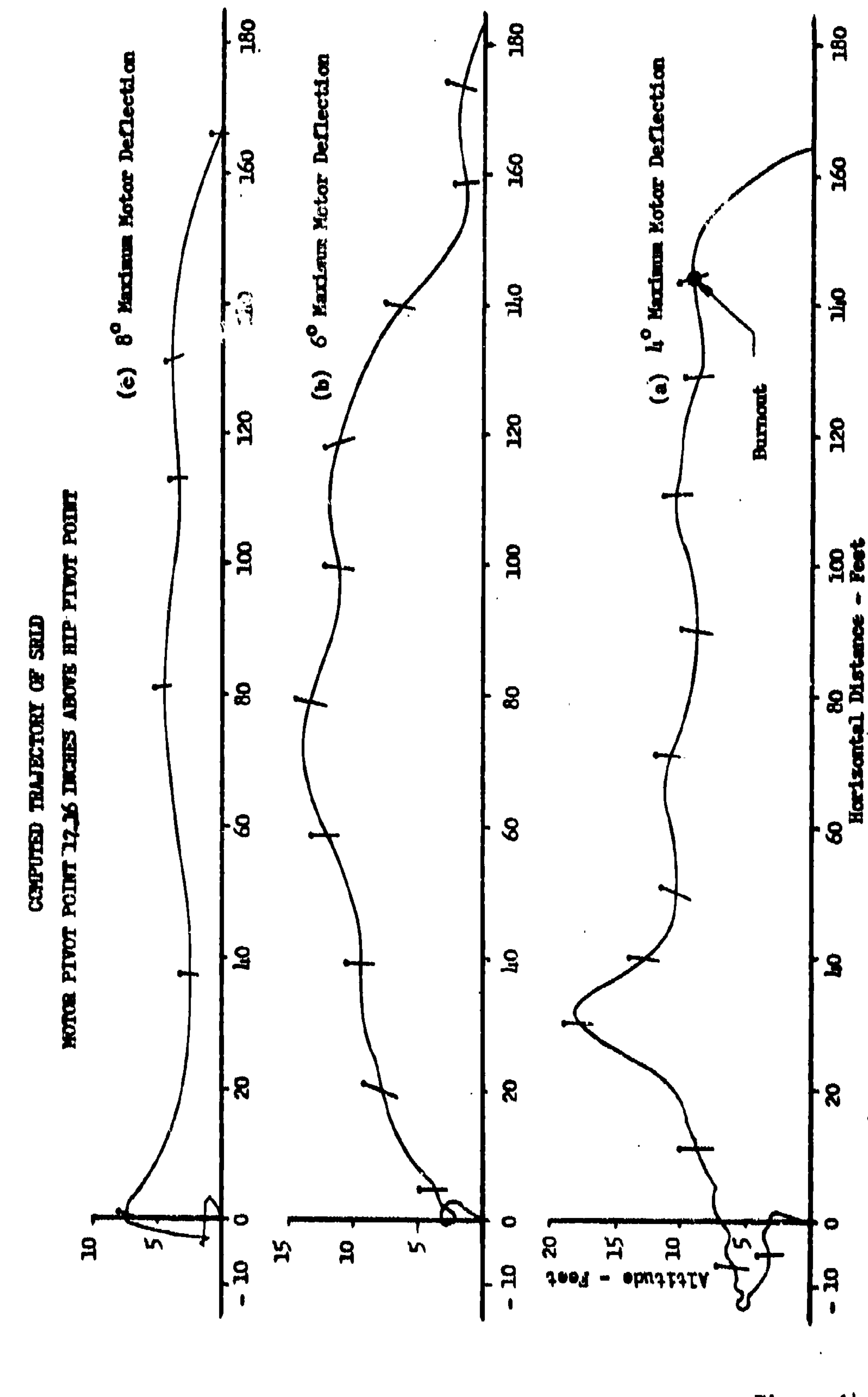

COMPUTED TRAJECTORY OF SRLD
MOTOR PIVOT POINT 12.36 INCHES ABOVE HIP PIVOT POINT
(c) 8° Maximum Motor Deflection
(b) 6° Maximum Motor Deflection
(a) 4° Maximum Motor Deflection
Burnout
Altitude - Feet
Horizontal Distance - Feet

Figure 14

COMPUTED TRAJECTORY OF SRLD

MOTOR PIVOT POINT 12.0 INCHES ABOVE HIP PIVOT POINT

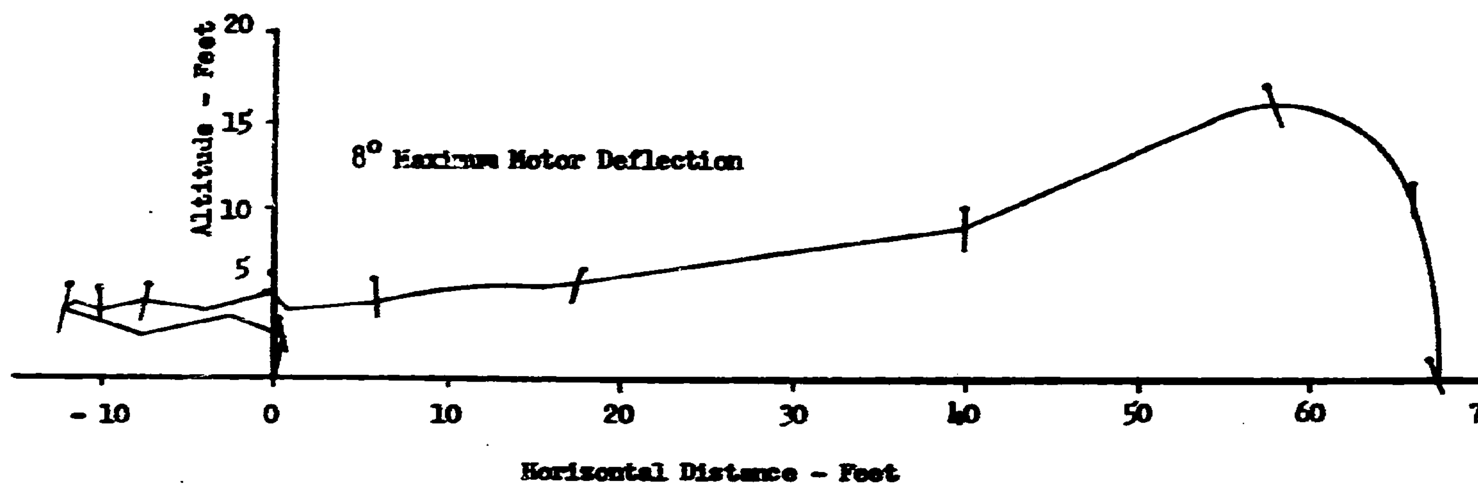


Figure 15

COMPUTED TRAJECTORY OF SRLD

MOTOR PIVOT POINT 19.2 INCHES ABOVE HIP PIVOT POINT

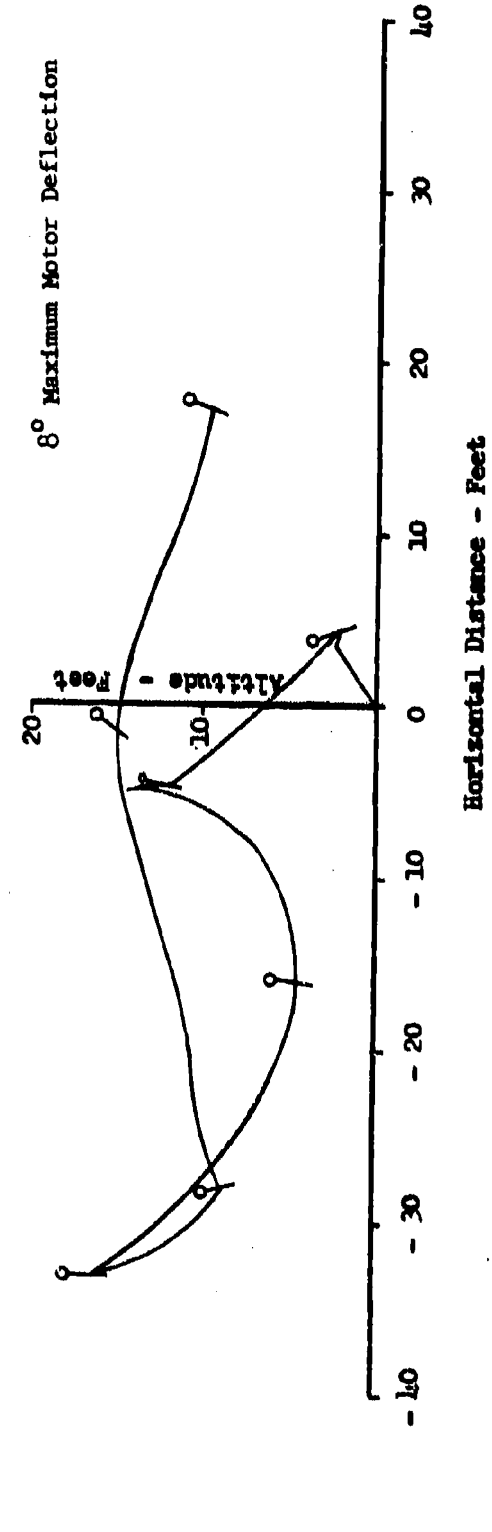


Figure 16

COMPUTED TRAJECTORY OF SLED
8° Maximum Motor Deflection

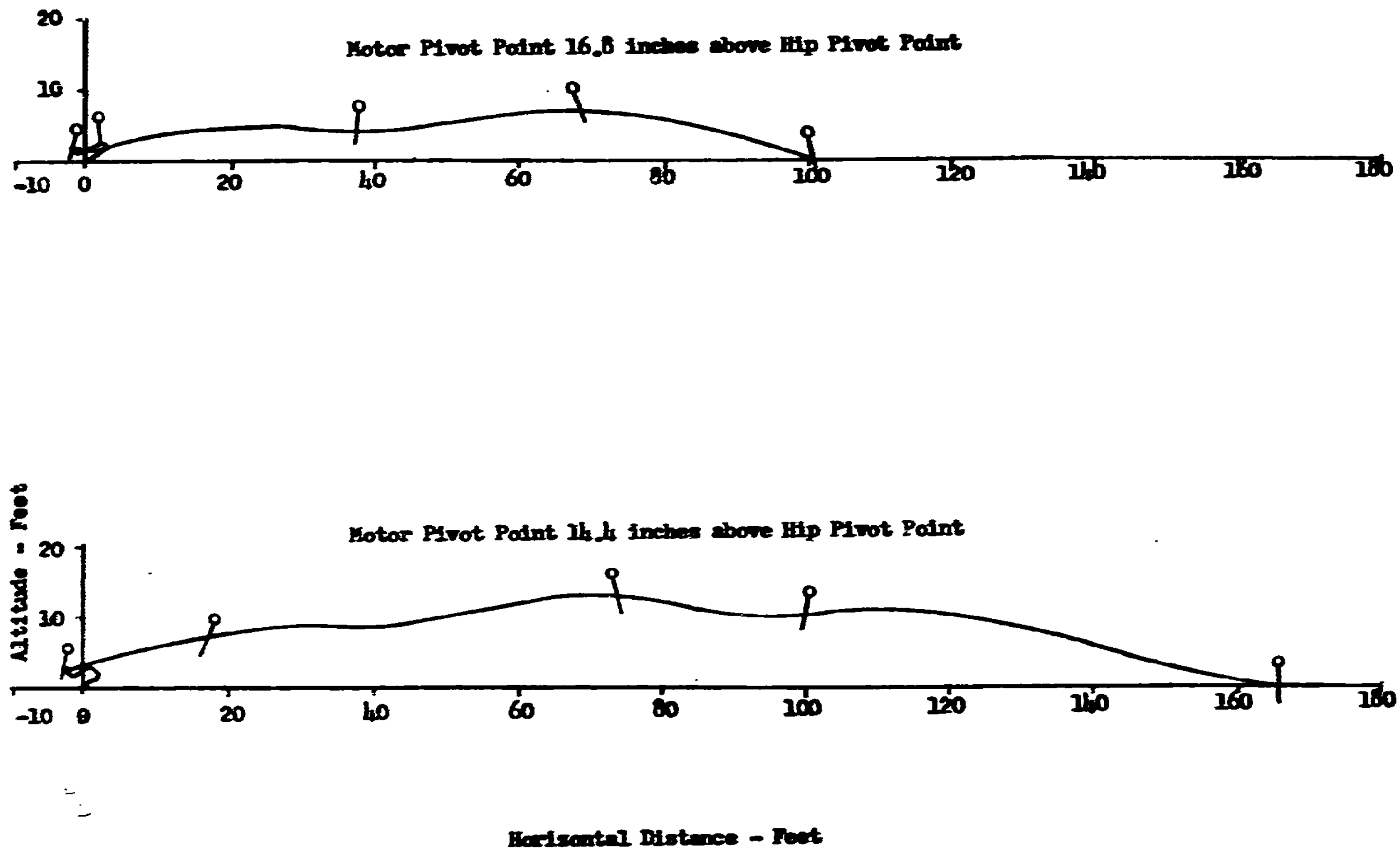


Figure 17

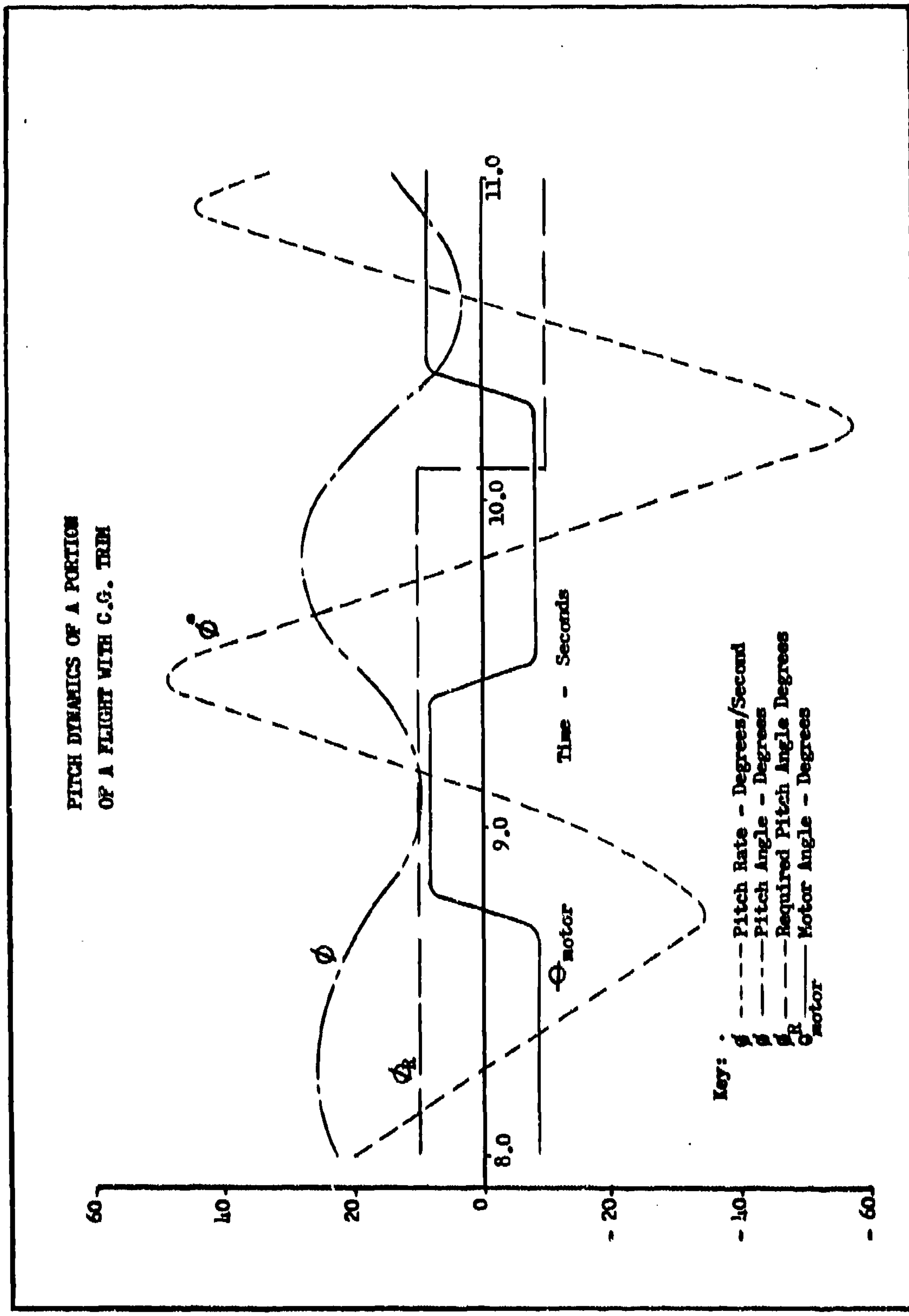

PITCH DYNAMICS OF A PORTION
OF A FLIGHT WITH C.G. TRIM
60
40
20
0
- 20
- 40
- 60
8.0
9.0
10.0
11.0
Time - Seconds
$\dot{\phi}$
ϕ
ϕ_R
θ_{motor}
Key:
$\dot{\phi}$ - - - Pitch Rate - Degrees/Second
ϕ — - — Pitch Angle - Degrees
ϕ_R — — — Required Pitch Angle Degrees
θ_{motor} ——— Motor Angle - Degrees

Figure 18

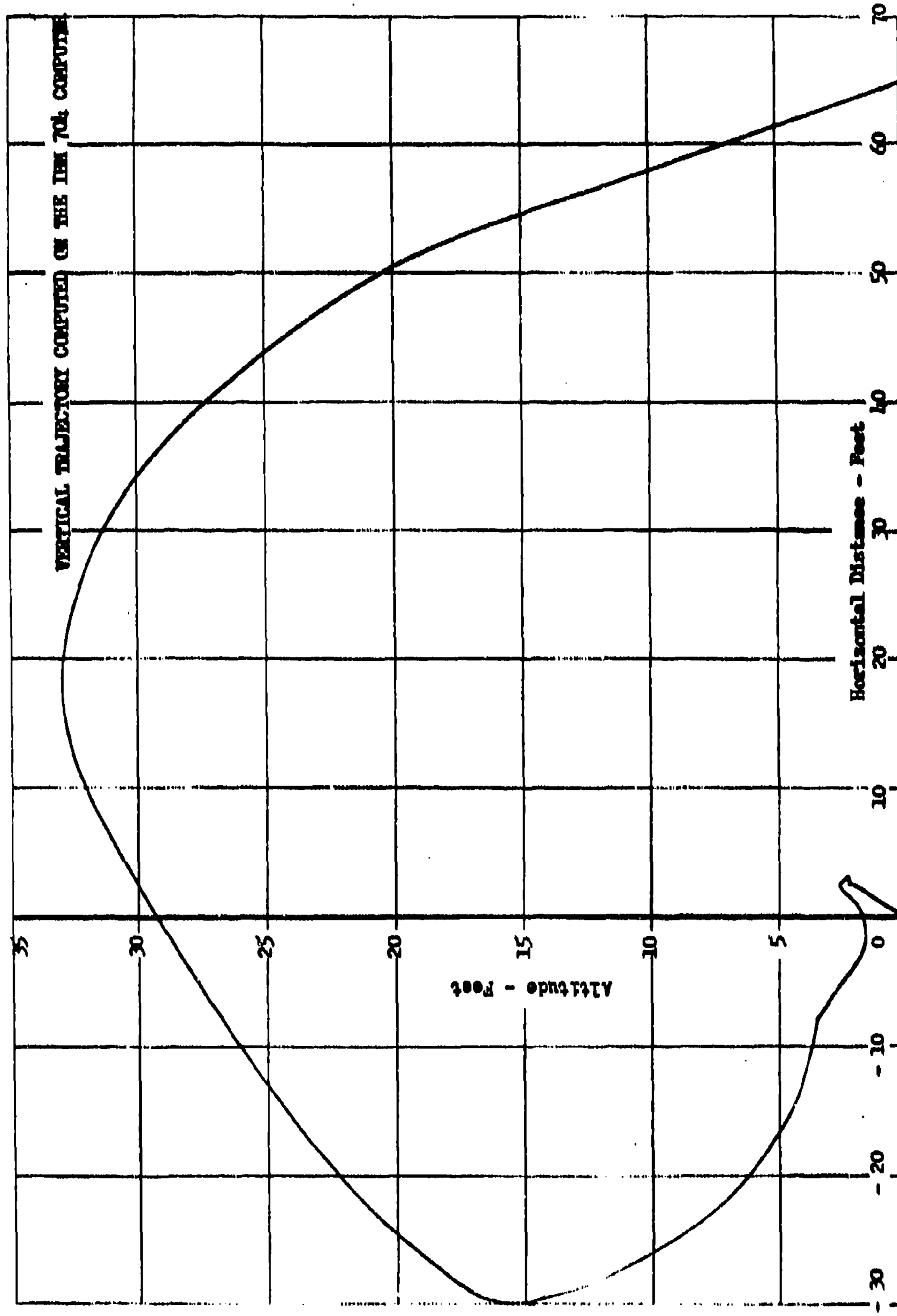

VERTICAL TRAJECTORY COMPUTED ON THE IBM 704 COMPUTER
Altitude - Feet
Horizontal Distance - Feet
35
30
25
20
15
10
5
0
70
60
50
40
30
20
10
0
-10
-20
-30

Figure 19

Kick Angle
vs
Duration of Kick
For 90° Kick

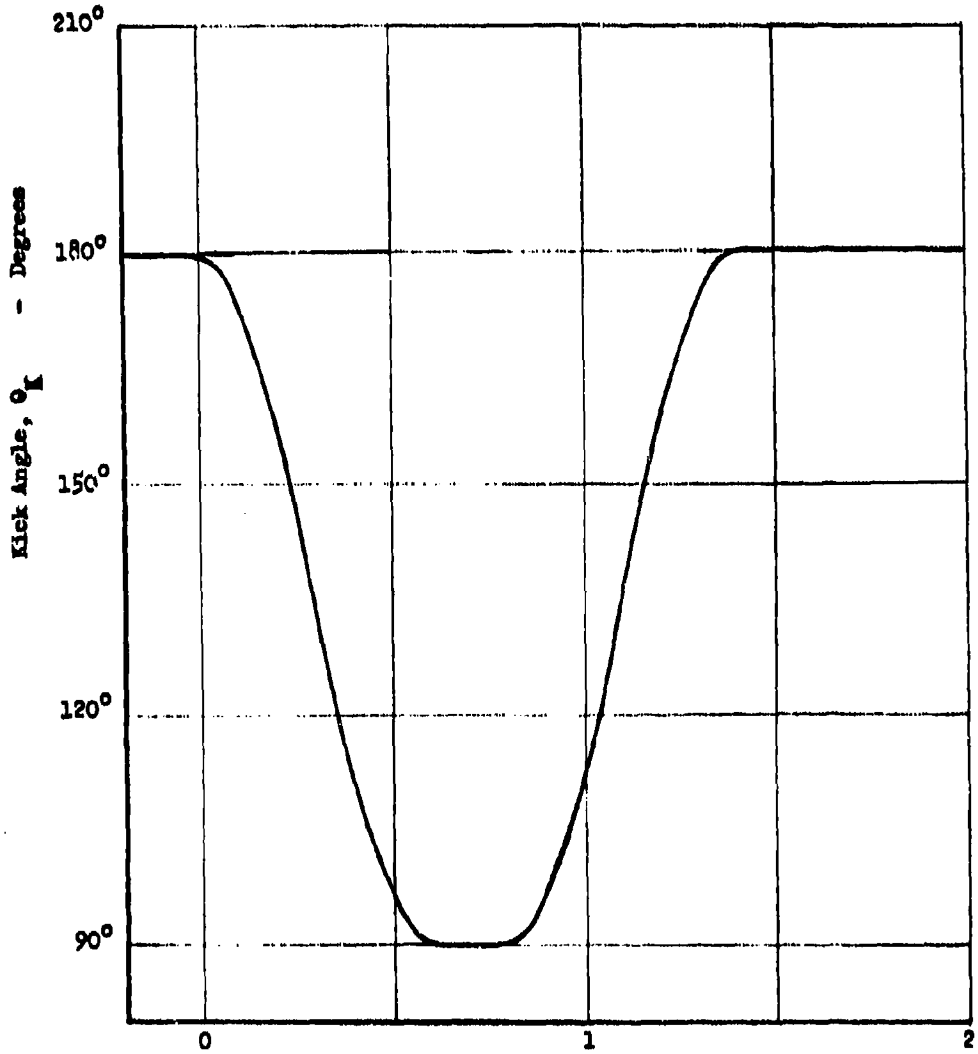


Figure 20

FLIGHT ATTITUDE DURING A KICK MANEUVER
FOR 90° KICK

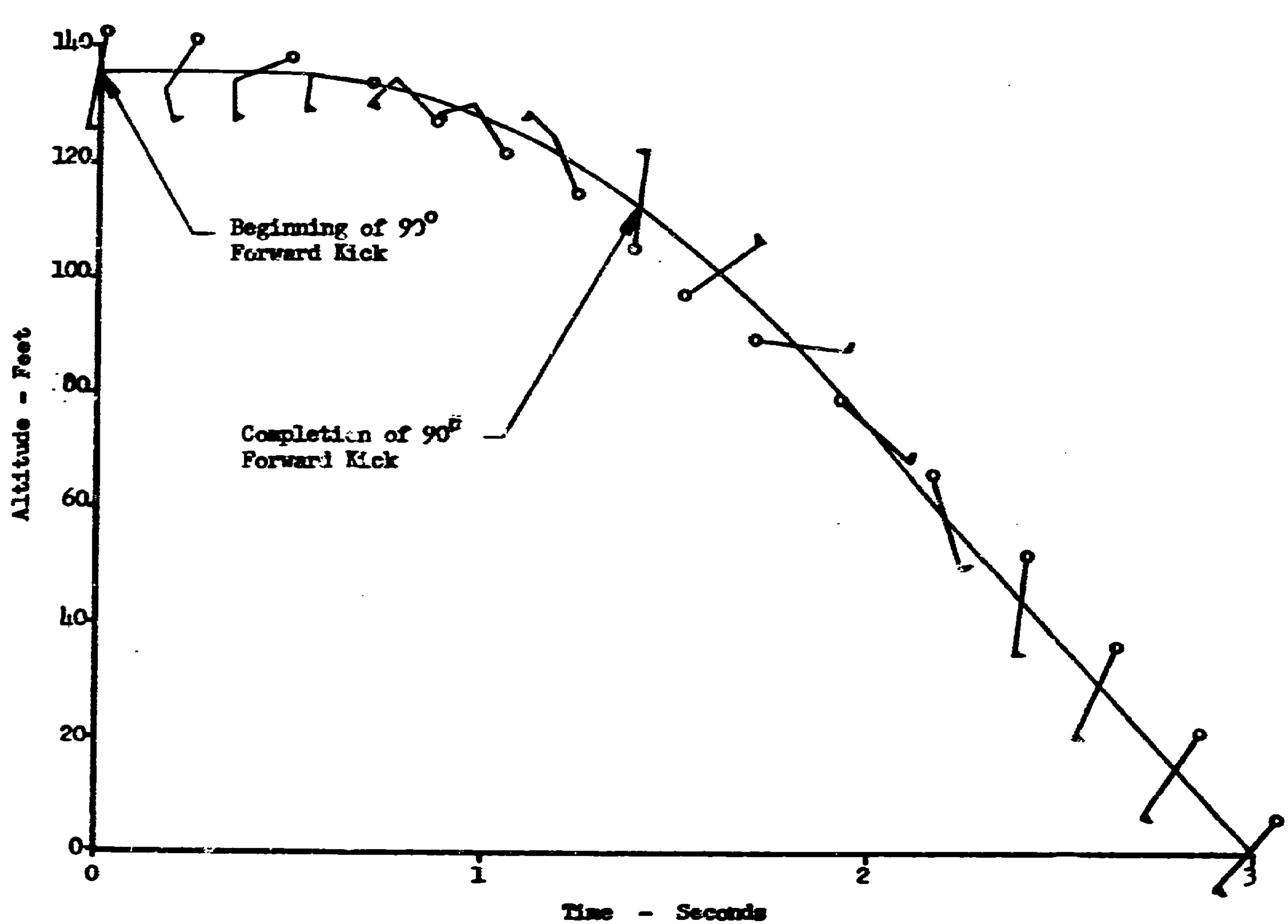


Figure 21

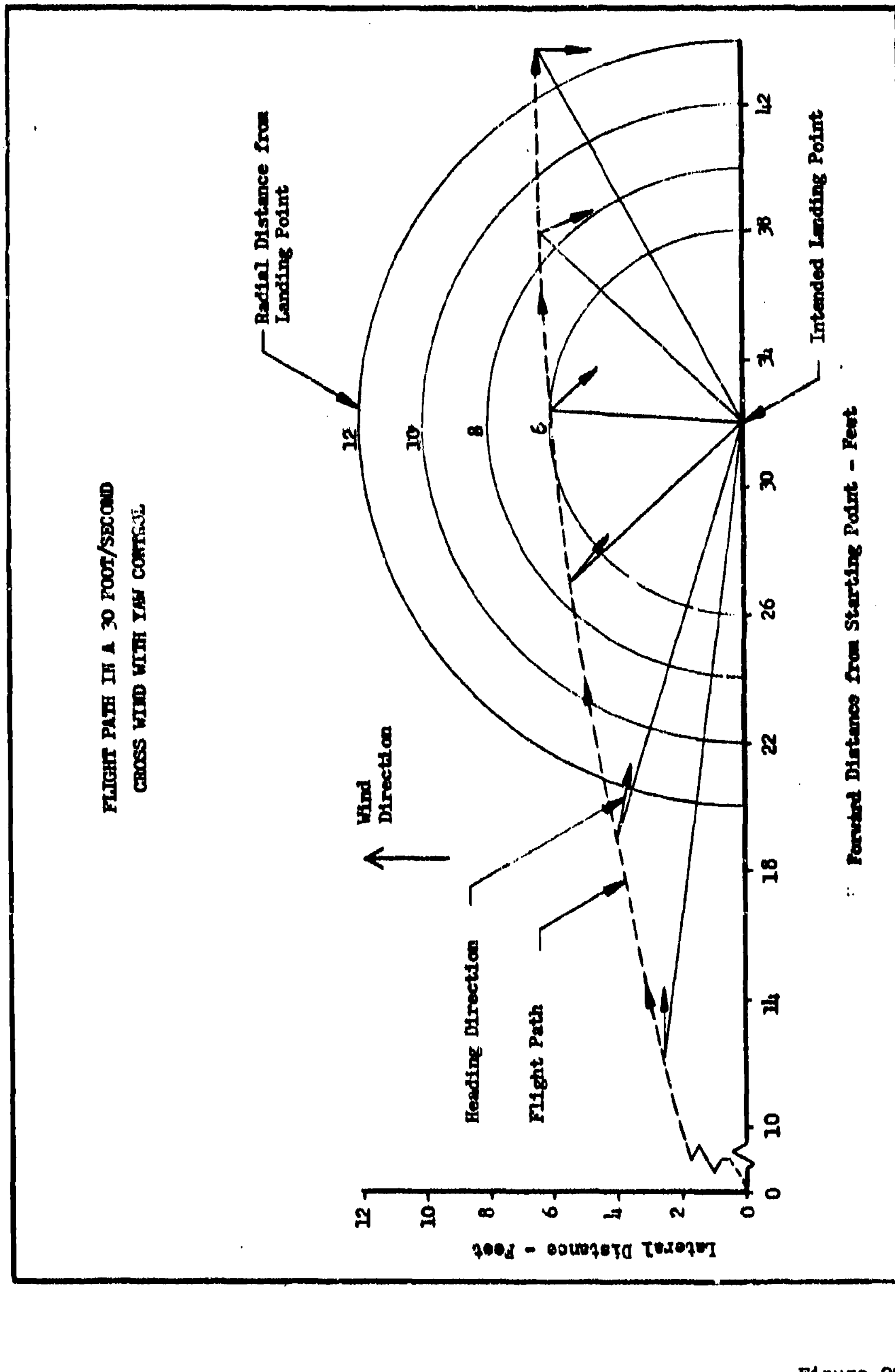
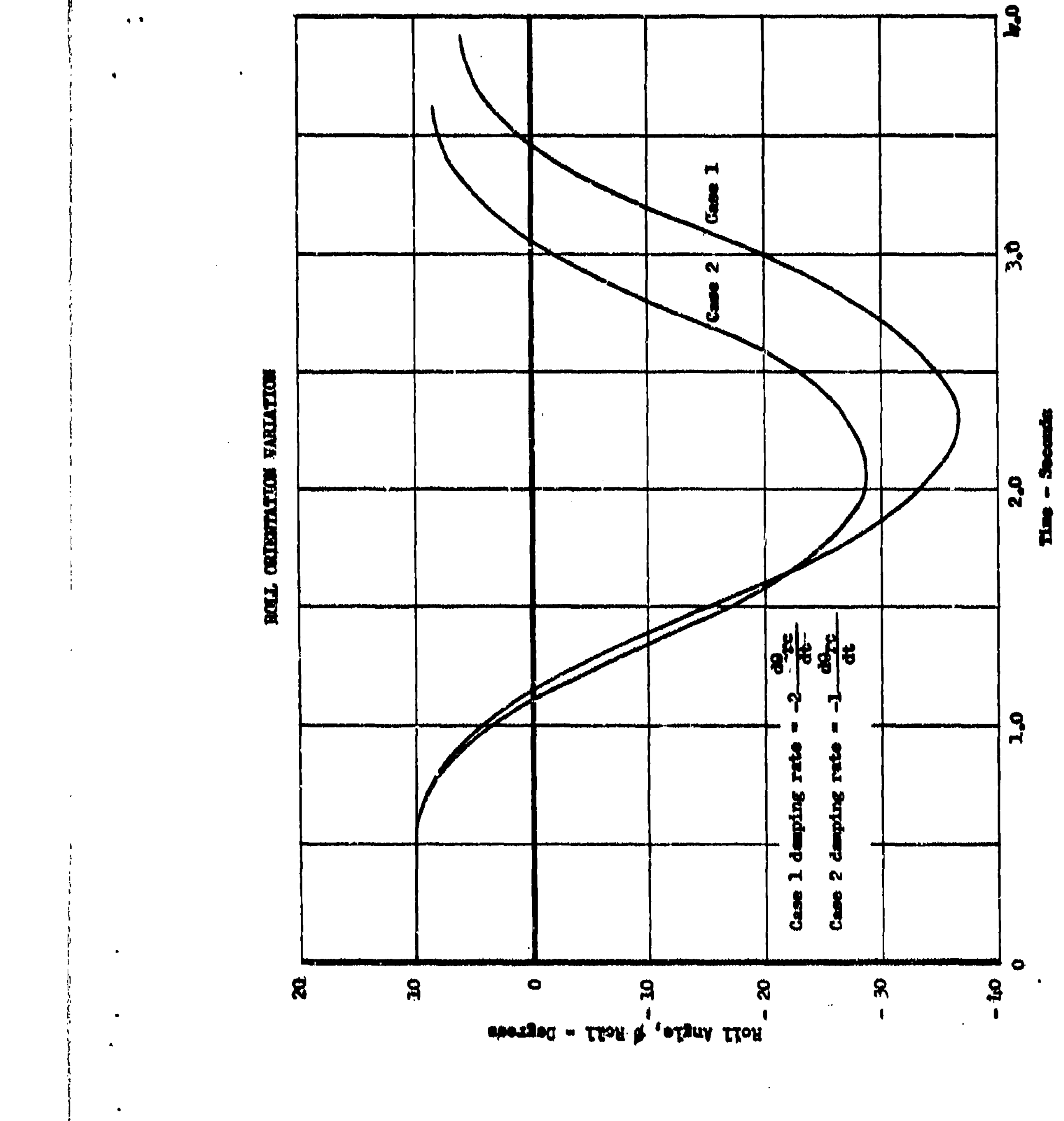

FLIGHT PATH IN A 30 FOOT/SECOND
CROSS WIND WITH YAW CONTROL
Radial Distance from Landing Point
Intended Landing Point
Wind Direction
Heading Direction
Flight Path
Forward Distance from Starting Point - Feet
Lateral Distance - Feet

Figure 22



ROLL ORIENTATION VARIATION
Roll Angle, φ Roll - Degrees
20
10
0
- 10
- 20
- 30
- 40
Time - Seconds
0
1.0
2.0
3.0
4.0
Case 1
Case 2
Case 1 damping rate = $-2 \frac{d\theta_{rc}}{dt}$
Case 2 damping rate = $-1 \frac{d\theta_{rc}}{dt}$

Figure 23

THE DEVELOPMENT OF THE EQUATIONS

OF MOTION FOR THE SMALL ROCKET LIFT DEVICE

DYNAMIC STABILITY STUDY

I. SUMMARY

The equations of motion for the SRLD maneuvering in the pitch plane are developed herein. The system is treated as a quasi-rigid body composed of two rigid components hinged together. The components represent the upper and lower parts of the operator's body and the hinge represents the hip joint.

Since it is intended that these equations be used for a digital computer study, several approximations have been made and finite difference solutions are sometimes used.

II. DISCUSSION

The requirement for a dynamic stability study of the SRLD presented several problems associated with the methods of solution to be used. The complexity of the system made it necessary that a digital computer be employed, rather than an analog machine. The ability of the system to hover, fly backwards, and change configuration made it impossible to employ any of the trajectory routines previously programmed for the IBM 704. Accordingly, a new set of equations of motion had to be developed for use with the digital machine.

After a preliminary investigation of different approaches, it was determined that the contract requirements could be most expeditiously satisfied by performing a stability and performance study of pitch plane motion and separate stability studies of roll and yaw motion. The two dimensional equations of motion in the pitch plane are developed in this report.

It is desired that the effect of a violent contortion of the operator's body on the controllability of the SRLD be investigated. The most violent contortion apparent, in the pitch plane, is the kicking of the legs from the hip. Accordingly, provision will be made for the inclusion of this motion at an arbitrary time during flight. Since it is expected that the operator will attempt to maintain a "standing-type" position during flight, the kick will consist of a kick forward from the "standing-type" position, a period during which the legs are held forward, and a kick down to the initial position. The mathematical model of the operator will assume two rigid bodies (Figure 1) of masses M_1 and M_2, pivoted at the hip, with the respective C.G.'s at distances r_1 and r_2 from the hip. The subscript ($_1$) will refer to the upper body including the SRLD and propellant, and the subscript ($_2$) will refer to the legs. A reference axis has been chosen as the line vertically upward from the hip pivot when the operator is in the standing position. It is assumed that the C.G. of the leg is on this line in the standing position. The angle between the r_2 line and the reference line will be called θ. The angle between the r_1 line and the reference line will be called γ.

A mathematical expression for the kick maneuver is represented by a sinusoidal velocity curve in the form:

$$\dot{\theta} = p \sin St' \qquad \text{for } t' \text{ from } 0 \text{ to } 2\,\Delta t$$

The corresponding angle and acceleration equations are:

$$\theta = \theta_0 + \frac{P}{S}(1 - \cos St')$$

$$\ddot{\theta} = PS \cos St'$$

A kick magnitude of 90° will be considered.

The time increment Δt will represent the time required for a 90° kick in one direction and t' will represent the total time during which kicking has been occurring. During the course of a kick up and back, t' will go from 0 to $2\,\Delta t$.

.. Curves of the functions of time (t) vs θ, $\dot{\theta}$, $\ddot{\theta}$ will be entered into the computer.
S is defined as $\pi/\Delta t$.
The maximum angular velocity P is chosen so that a 90° kick is accomplished during Δt.

In these computations, Δt was assumed to be approximately 0.6 seconds.

The corresponding value of S is 1.667π and the value of P is $-.439\ \pi^2$.

θ' is defined as $\theta + \gamma$

however, since:

$\dot{\theta} \gg \dot{\gamma}$

$\ddot{\theta} \gg \ddot{\gamma}$ (except during impulse thrust changes which, having 0 duration, can be shown not to affect the dynamics of the problem.)

We define:

$\dot{\theta}' = \dot{\theta}$

$\ddot{\theta}' = \ddot{\theta}$

Since the kick is accomplished by applying equal and opposite torques to the two body components, there will be equal and opposite changes in the angular momentum of the two. It will be assumed that the thrust and aerodynamic moments do not change θ. Hence, the total internal angular momentum will remain unchanged. We may therefore look at the system as if it were a rigid body with variable moment of inertia about its C.G. Considering the two components to be rigidly coupled masses, we may write:

$$I_c = I_1 + I_2 + a^2 M_1 + b^2 M_2$$

The angular velocity of the system, ω, is given, for a rigid body, as the angular velocity of any line on the body (two dimensional). Since we have a quasi-rigid body, the generalization is not necessarily valid. The line through the C.G.'s 1, 2, and C, however, is a special case which, for the quasi-rigid body, has the same properties as the arbitrary line on the rigid body. This line makes an angle η with coordinate y of a non-rotating axis system. Therefore:

$$\omega = \dot{\eta}$$

The objective of the kick routine is to determine the angular inclination, ϕ, of the upper body reference axis from the vertical. Since the position of the body reference axis will be defined relative to the position of the upper body C.G. by a curve, it is convenient to follow the inclination of the line connecting the pivot point with the upper C.G. (line r_1). We will designate the angle between this line and the y axis as ϕ'. ξ defined as the angle between r_1 and the line connecting the pivot point with the combined C.G. (line r_c).

We may now write the expressions relating the torque and the angular momentum:

$$I_c \omega = \left[I_1 + I_2 + a^2 M_1 + b^2 M_2\right] \dot{\eta}$$

$$\frac{d}{dt}\left(I_c \omega\right) = \tau_{EXTERNAL} = EXTERNAL\ TORQUE$$

or

$$\left[I_1 + I_2 + a^2 M_1 + b M_2\right] \ddot{\eta} + \left[\dot{I}_1 + \dot{I}_2 + M_1 \frac{d}{dt}(a^2) + M_2 \frac{d}{dt}(b^2) + \frac{a^2 \dot{w}}{g}\right] \dot{\eta} = \tau_{EXTERNAL}$$

However, from Figure 2:

$$\eta = \phi' - \eta'$$

so that,

$$I_c\left(\ddot{\phi}' - \ddot{\eta}'\right) + \dot{I}_c\left(\dot{\phi}' - \dot{\eta}'\right) = \tau$$

or:

$$\ddot{\phi}' = \ddot{\eta}' + \frac{\tau + \dot{I}_c\left(\dot{\eta}' - \dot{\phi}'\right)}{I_c}$$

a^2 and b^2 may be simply determined:

$$(a + b)^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos \theta'$$

From the definition of C.G.:

$$a M_1 = b M_2$$

so that:

$$a^2 = \left(\frac{M_2}{M_c}\right)^2 \left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos \theta'\right)$$

$$b^2 = \left(\frac{M_1}{M_c}\right)^2 \left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos \theta'\right)$$

We must next determine expressions for $\dot{\eta}'$ and $\ddot{\eta}'$. Since r_1, r_2 and θ' will be known or determinable, η' is defined in terms of these quantities:

$$\eta' = \tan^{-1} \frac{r_2 \sin\theta'}{r_1 - r_2 \cos\theta'}$$

Differentiating with respect to time yields:

$$\dot{\eta}' = \frac{(r_1 - r_2\cos\theta')(r_2\dot{\theta}'\cos\theta' + \dot{r}_2\sin\theta') - r_2\sin\theta'(\dot{r}_1 - \dot{r}_2\cos\theta' + r_2\dot{\theta}'\sin\theta')}{\left[1 + \left(\dfrac{r_2\sin\theta'}{r_1 - r_2\cos\theta'}\right)^2\right](r_1 - r_2\cos\theta')}$$

$\dot{r}_2$, the rate of change of the distance from the hip pivot to the leg C.G. is zero, so terms containing this may be eliminated. Combining and rearranging the remaining terms yields:

$$\dot{\eta}' = \frac{r_1 r_2\dot{\theta}'\cos\theta' - r_2\dot{r}_1\sin\theta' - r_2^2\dot{\theta}'}{r_1^2 + r_2^2 - 2r_1 r_2\cos\theta'}$$

Again differentiating with respect to time yields:

$$\ddot{\eta}' = \Big\{\left(r_1^2 + r_2^2 - 2r_1 r_2\cos\theta'\right)\Big[\dot{r}_1 r_2\dot{\theta}'\cos\theta' + r_1\dot{r}_2\dot{\theta}'\cos\theta' + r_1 r_2\ddot{\theta}'\cos\cos\theta' -$$

$$r_1 r_2(\dot{\theta}')^2\sin\theta' - \dot{r}_2\dot{r}_1\sin\theta' - r_2\ddot{r}_1\sin\theta' - r_2\dot{r}_1\dot{\theta}'\cos\theta' - 2r_2\dot{r}_2\dot{\theta}' - r_2^2\ddot{\theta}'\Big] -$$

$$\left(r_1 r_2\dot{\theta}'\cos\theta' - r_2\dot{r}_1\sin\theta' - r_2^2\dot{\theta}'\right)\Big[2r_1\dot{r}_1 + 2r_2\dot{r}_2 - 2\dot{r}_1 r_2\cos\theta' -$$

$$2r_1\dot{r}_2\cos\theta' + 2r_1 r_2\dot{\theta}'\sin\theta'\Big]\Big\} \Big/ \left(r_1^2 + r_2^2 - 2r_1 r_2\cos\theta'\right)^2$$

Again, terms containing $\dot{r}_2$ will be eliminated. The position of the upper body C.G. is taken to vary linearly with propellant expended. Since the total C.G. shift is on the order of only 1 1/2 inches during 15 seconds of burning time, $\dot{r}_1$ is quite small, and $\ddot{r}_1$ is negligible. Therefore, we may eliminate terms containing $\ddot{r}_1$. Combining and rearranging the remaining terms yields:

$$\ddot{\eta}' = \frac{r_1 r_2 (\ddot{\theta}' \cos\theta' - \dot{\theta}'^2 \sin\theta') - r_2^2 \ddot{\theta}'}{(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta')} - \frac{2 r_2^2}{(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta')^2} \Bigg[$$

$$\frac{r_1^2 \dot{r}_1}{r_2} \dot{\theta}' \cos\theta' - r_1 \dot{\theta}' \dot{r}_1 \cos^2\theta' + r_1^2 \dot{\theta}'^2 \sin\theta' \cos\theta' + \dot{r}_1^2 \sin\theta' \cos\theta' -$$

$$\frac{r_1}{r_2} \dot{r}_1^2 \sin\theta' - r_1 \dot{r}_1 \dot{\theta}' \sin^2\theta' - r_1 \dot{r}_1 \dot{\theta}' + r_2 \dot{r}_1 \dot{\theta}' \cos\theta' - r_2 r_1 \dot{\theta}'^2 \sin\theta' \Bigg]$$

Inspecting the terms in $\dot{\eta}'$ and $\ddot{\eta}'$, it is apparent that many of them contain $\dot{r}_1$. The significance of these terms is that they represent the motion of the body due to the rate of shift of the fuel C.G. at constant mass. It is apparent that the effect of these terms is small compared with the effect of $\ddot{\theta}'$ and $\dot{\theta}'$ and a great deal of simplification results if they may be discarded. This is now done and we get:

$$\dot{\eta}' = \frac{r_1 r_2 \dot{\theta}' \cos\theta' - r_2^2 \dot{\theta}'}{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'}$$

$$\ddot{\eta}' = \frac{r_1 r_2\left(\ddot{\theta}'\cos\theta' - \dot{\theta}'^2 \sin\theta'\right) - r_2^2\,\ddot{\theta}'}{\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)} - \frac{2 r_1^2 r_2^2\,\dot{\theta}'^2 \sin\theta' \cos\theta' - 2 r_2^3 r_1\,\dot{\theta}'^2 \sin\theta'}{\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)^2}$$

The next items to be determined are expressions for $\frac{d}{dt}(a^2)$ and $\frac{d}{dt}(b^2)$

$$\frac{d}{dt}(a^2) = \left[\left(\frac{M_2}{M_c}\right)^2 \frac{d}{dt}\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)\right] + \left[\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)\frac{d}{dt}\left(\frac{M_2}{M_c}\right)^2\right]$$

$$\frac{d}{dt}(b^2) = \left[\left(\frac{M_1}{M_c}\right)^2 \frac{d}{dt}\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)\right] + \left[\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)\frac{d}{dt}\left(\frac{M_1}{M_c}\right)^2\right]$$

$$\frac{d}{dt}\left[\frac{M_2}{M_c}\right]^2 = 2\,\frac{\left(M_c \dot{M}_2 - M_2 \dot{M}_c\right) M_2}{M_c^3}$$

However,

$$\dot{M}_2 = 0,$$

and $\dot{M}_c$ may be written as $\dot{w}/g$

Therefore:

$$\frac{d}{dt}\left[\frac{M_2}{M_c}\right]^2 = -2\,\frac{M_2^2\,\dot{w}}{g\,M_c^3}$$

$$\frac{d}{dt}\left[\frac{M_1}{M_c}\right]^2 = 2\,\frac{\left(M_c \dot{M}_1 - M_1 \dot{M}_c\right) M_1}{M_c^3}$$

but,

$$\dot{M}_1 = \dot{M}_c = \dot{w}\, g$$

and,

$$M_c - M_1 = M_2$$

Therefore:

$$\frac{d}{dt}\left[\frac{M_1}{M_c}\right]^2 = 2\,\frac{M_1 M_2 \dot{w}}{g\, M_c^3}$$

$$\frac{d}{dt}\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right) = 2 r_1 \dot{r}_1 + 2 r_2 \dot{r}_2 - 2 \dot{r}_1 r_2 \cos\theta' - 2 r_1 \dot{r}_2 \cos\theta' + 2 r_1 r_2 \dot{\theta}' \sin\theta'$$

Since $r_2 = 0$, this reduces to:

$$\frac{d}{dt}\left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right] = 2\left(r_1 \dot{r}_1 - \dot{r}_1 r_2 \cos\theta' + r_1 r_2 \dot{\theta}' \sin\theta'\right)$$

Therefore:

$$\frac{d}{dt}(a^2) = 2\left(\frac{M_2}{M_c}\right)^2\left(r_1 \dot{r}_1 - \dot{r}_1 r_2 \cos\theta' + r_1 r_2 \dot{\theta}' \sin\theta'\right) - 2\,\frac{M_2^2 \dot{w}}{g\, M_c^3}\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)$$

$$\frac{d}{dt}(b^2) = 2\left(\frac{M_1}{M_c}\right)^2\left(r_1 \dot{r}_1 - \dot{r}_1 r_2 \cos\theta' + r_1 r_2 \dot{\theta}' \sin\theta'\right) + 2\,\frac{M_1 M_2 \dot{w}}{g\, M_c^3}\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)$$

Since the change in (a) due to $\dot{r}_1$ is small compared to the change due to θ' during the interval of the kick, terms containing $\dot{r}_1$ are eliminated, and we are left with:

$$\frac{d}{dt}(a^2) = \frac{2 M_2^2 r_1 r_2 \dot{\theta}' \sin\theta'}{M_c^2} - \frac{2 M_2^2 \dot{w}\left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)}{g\, M_c^3}$$

$$\frac{d}{dt}\left(b^2\right) = \frac{2 M_1^2 r_1 r_2 \dot{\theta}' \sin\theta'}{M_c^2} + \frac{2 M_1 M_2 \dot{w} \left(r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta'\right)}{g M_c^3}$$

$\dot{I}_1$ may be expressed as:

$$\frac{\dot{w}\, dI_1}{dw}$$

$\dot{I}_2$ is obviously 0.

We now repeat the pitching equation of motion

$$\left[I_1 + I_2 + a^2 M_1 + b^2 M_2\right]\ddot{\eta} + \left[\dot{I}_1 + \dot{I}_2 + M_1 \frac{d}{dt}\left(a^2\right) + M_2 \frac{d}{dt}\left(b^2\right) + \frac{a^2 \dot{w}}{g}\right]\dot{\eta} = \tau_{EXT}$$

It is apprent that we are now able to evaluate all the terms of the left hand member and it remains only to evaluate $\tau_{external}$ to allow the solution of the equation.

The external torque will be contributed by two items:

(1) Thrust
(2) Aerodynamic forces

The torque due to thrust will be evaluated next:

The thrust component parallel to the upper body reference axis will be F cos θ_M and that normal to the axis F sin θ_M. As may be seen from Figure 3, the resultant torque about the combined C.G. will be:

$$\tau_{THRUST} = \left[r_c \sin(\psi - \gamma) + x_M'\right] F \cos\theta_M + \left[y_M' - r_c \cos(\psi - \gamma)\right] F \sin\theta_M$$

And

$$r_c = \left\{\left(M_2 r_2 \sin\theta' / M_c\right)^2 + \left[\left(M_1 r_1 + M_2 r_2 \cos\theta'\right) / M_c\right]^2\right\}^{1/2}$$

$$\sin\psi = M_2 r_2 \sin\theta' / M_c r_c$$

We now proceed to evaluate the torque due to aerodynamic forces. Since the aerodynamic forces due to pitch damping will be small compared to those due to translation, we will ignore them. It will be further assumed that the aerodynamic forces may be broken into three flat plate drag components with little loss of accuracy. The components will act normal to the upper and lower body axes and parallel to the upper body axis. The drag forces are represented by D', and the negatives of these forces by D. The coefficients of the V^2 terms will be represented by f and are defined by:

$$f = \tfrac{1}{2} \rho C_D S$$

so that:

$$D = f\,V|V| = -D'$$

Figure 4 defines the three drag forces:

$$D_1 = f_1\left(V_Y \cos\phi + V_X' \sin\phi\right)\left|V_Y \cos\phi + V_X' \sin\phi\right| = -D_1'$$

$$D_2 = f_2\left(V_X' \cos\phi - V_Y \sin\phi\right)\left|V_X' \cos\phi - V_Y \sin\phi\right| = -D_2'$$

$$D_3 = f_3\left(V_X' \sin\beta + V_Y \cos\beta\right)\left|V_X' \sin\beta + V_Y \cos\beta\right| = -D_3'$$

$$\beta = \phi + \theta - \frac{\pi}{2}$$

$$\cos\beta = \sin\left(\theta + \phi\right)$$

$$\sin\beta = -\cos\left(\theta + \phi\right)$$

$$D_3 = f_3\left[-V_X' \cos(\theta+\phi) + V_Y \sin(\theta+\phi)\right]\left|-V_X' \cos(\theta+\phi) + V_Y \sin(\theta+\phi)\right|$$

As may be seen in Figure 5, the torque due to aerodynamic forces may be written as:

$$\Upsilon_{AERO} = D_1 \ell_1 + D_2 \ell_2 - D_3 \ell_3$$

or

$$\Upsilon_{AERO} = D_3' \ell_3 - D_1' \ell_1 - D_2' \ell_2$$

From Figure 4, it will be seen that l_1, l_2 and l_3 may be written in terms of previously defined quantities:

$$\ell_1 = X'_{OU} + r_c \, SIN(\psi - \gamma)$$

$$\ell_2 = Y'_{OU} - r_c \, COS(\psi - \gamma)$$

$$\ell_3 = r_4 - r_c \, COS(\theta' - \psi)$$

Thus, the torque due to aerodynamic forces is given by:

$$\Upsilon_{AERO} = -f_1 (V_Y \, COS\,\phi + V_X' \, SIN\,\phi) \left| V_Y \, COS\,\phi + V_X' \, SIN\,\phi \right| \left[X'_{OU} + r_c \, SIN(\psi - \gamma) \right] -$$

$$f_2 (V_X' \, COS\,\phi - V_Y \, SIN\,\phi) \left| V_X' \, COS\,\phi - V_Y \, SIN\,\phi \right| \left[Y'_{OU} - r_c \, COS(\psi - \gamma) \right] +$$

$$f_3 \left[-V_X' \, COS(\theta + \phi) + V_Y \, SIN(\theta + \phi) \right] \left| -V_X' \, COS(\theta + \phi) + V_Y \, SIN(\theta + \phi) \right| \left[r_4 - r_c \, COS(\theta' - \psi) \right]$$

We may now write the pitching equation of motion in terms of readily determinable quantities. We first define the torque equations in consistent terms.

The sum of the torques, due to thrust and aerodynamic forces are:

$$\tau_{EXTERNAL} = \left\{\left[r_c SIN(\psi-\gamma)+X_M'\right]F COS\,\theta_M\right\} + \left\{\left[Y_M' - r_c COS(\psi-\gamma)\right]F SIN\,\theta_M\right\} - \left\{f_1\left(V_Y COS\,\phi + V_X' SIN\,\phi\right)\left|V_Y COS\,\phi + V_X' SIN\,\phi\right|\left[X_{DU}' + r_c SIN(\psi-\gamma)\right]\right\} - \left\{f_2\left(V_X' COS\,\phi - V_Y SIN\,\phi\right)\left|V_X' COS\,\phi - V_Y SIN\,\phi\right|\left[Y_{DU}' - r_c COS(\psi-\gamma)\right]\right\} + \left\{f_3\left[-V_X' COS(\theta+\phi) + V_Y SIN(\theta+\phi)\right]\left|-V_X' COS(\theta+\phi) + V_Y SIN(\theta+\phi)\right|\left[r_4 - r_c COS(\theta'-\psi)\right]\right\}$$

where:

$$r_c = \left\{\left(M_2 r_2 SIN\,\theta'/M_c\right)^2 + \left[\left(M_1 r_1 + M_2 r_2 COS\,\theta'\right)/M_c\right]^2\right\}^{1/2}$$

$$r_c = \frac{\left\{\left[M_2 r_2\ SIN(\theta+\gamma)\right]^2 + \left[M_1 r_1 + M_2 r_2 COS(\theta+\gamma)\right]^2\right\}^{1/2}}{M_c}$$

and,

$$SIN\,\psi = \frac{M_2 r_2}{M_c r_c} SIN(\theta+\gamma)$$

however:

$$\theta + \phi = \theta' + \phi'$$

$$\theta' = \theta + \gamma$$

$$V_X' = V_X + V_W$$

so that:

$$\tau_{EXT} = \left\{\left[r_c \sin(\psi-\gamma) + X'_M\right] F \cos\theta_M\right\} + \left\{\left[Y'_M - r_c \cos(\psi-\gamma)\right] F \sin\theta_M\right\} - \left\{f_1 \left[V_Y \cos\phi + (V_x + V_w) \sin\phi\right] \left|V_Y \cos\phi + (V_x+V_w)\sin\phi\right| \left[X'_{OU} + r_c \sin(\psi-\gamma)\right]\right\} - \left\{f_2 \left[(V_x+V_w)\cos\phi - V_Y \sin\phi\right] \left|(V_x+V_w)\cos\phi - V_Y \sin\phi\right| \left[Y'_{OU} - r_c \cos(\psi-\gamma)\right]\right\} + \left\{f_3 \left[-(V_x+V_w)\cos(\theta+\phi) + V_Y \sin(\theta+\phi)\right] \left|-(V_x+V_w)\cos(\theta+\phi) + V_Y \sin(\theta+\phi)\right| \left[r_4 - r_c \cos(\theta+\gamma-\psi)\right]\right\}$$

The pitch equation of motion will be used in the form:

$$\ddot{\phi}' = \ddot{\eta}' + \frac{1}{I_c}\left[\tau + \dot{I}_c\left(\dot{\eta}' - \dot{\phi}'\right)\right]$$

Writing I_c in the units used for τ:

$$I_c = I_1 + I_2 + \left[M_1\left(\frac{M_2}{M_c}\right)^2 + M_2\left(\frac{M_1}{M_c}\right)^2\right]\left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta+\gamma)\right]$$

Writing $\dot{I}_c$ in the same units:

$$\dot{I}_c = \frac{dI_1}{dw}\dot{w} + 2M_1\left[\frac{M_2^2 r_1 r_2 \dot{\theta} \sin(\theta+\gamma)}{M_c^2} - \frac{M_2^2 \dot{w} \left\{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta+\gamma)\right\}}{g M_c^3}\right] +$$

$$2M_2\left[\frac{M_1^2 r_1 r_2 \dot{\theta} \sin(\theta+\gamma)}{M_c^2} + \frac{M_1 M_2 \dot{w} \left\{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta+\gamma)\right\}}{g M_c^3}\right] +$$

$$\left(\frac{M_2}{M_c}\right)^2 \frac{\dot{w}\left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta+\gamma)\right]}{g}$$

writing $\dot{\eta}'$:

$$\dot{\eta}' = \frac{r_1 r_2 \dot{\theta} \cos(\theta + \gamma) - r_2^2 \dot{\theta}}{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)}$$

writing $\ddot{\eta}'$:

$$\ddot{\eta}' = \frac{r_1 r_2 \left[\ddot{\theta} \cos(\theta + \gamma) - \dot{\theta}^2 \sin(\theta + \gamma)\right] - r_2^2 \ddot{\theta}}{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)} - \frac{2 r_1^2 r_2^2 \dot{\theta}^2 \sin(\theta + \gamma) \cos(\theta + \gamma) - 2 r_2^3 r_1 \dot{\theta}^2 \sin(\theta + \gamma)}{\left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)\right]^2}$$

We may write:

$$\phi' = \phi - \gamma$$
$$\dot{\phi}' = \dot{\phi} - \dot{\gamma}$$
$$\ddot{\phi}' = \ddot{\phi} - \ddot{\gamma}$$

However, as mentioned earlier, $\ddot{\gamma}$ may be ignored compared with $\ddot{\phi}$, so that

$$\ddot{\phi}' = \ddot{\phi}$$

also:

$$\dot{\gamma} = \frac{d\gamma}{dw} \dot{w}$$

so that:

$$\dot{\phi}' = \dot{\phi} - \frac{d\gamma}{dw} \dot{w}$$

We finally are able to write the pitching equation of motion:

$$\ddot{\phi} = \left\{ \frac{r_1 r_2 \left[\ddot{\theta} \cos(\theta + \gamma) - \dot{\theta}^2 \sin(\theta + \gamma)\right] - r_2^2 \ddot{\theta}}{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)} \right\} - \left\{ \frac{2 r_1^2 r_2^2 \dot{\theta}^2 \left[\sin(\theta + \gamma)\right]\left[\cos(\theta + \gamma)\right] - 2 r_2^3 r_1 \dot{\theta}^2 \sin(\theta + \gamma)}{\left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)\right]^2} \right\} +$$

$$\left\{ \frac{1}{I_1 + I_2 + \left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)\right]\left[M_1 \left(\frac{M_2}{M_c}\right)^2 + M_2 \left(\frac{M_1}{M_c}\right)^2\right]} \right\} \Bigg\{ \Bigg[\Big[\Big\{ \left[r_c \sin(\psi - \gamma) + X_M'\right] F \cos \theta_M \Big\} +$$

$$\left[Y_M' - r_c \cos(\psi - \gamma)\right] F \sin \theta_M \Big\} - \Big\{ f_1 \left[V_Y \cos \phi + (V_x + V_w) \sin \phi\right] \Big| V_Y \cos \phi + (V_x + V_w) \sin \phi \Big| \left[X_{bu}' + r_c \sin(\psi - \gamma)\right] \Big\} -$$

$$\Big\{ f_2 \left[(V_Y + V_w) \cos \phi - V_Y \sin \phi\right] \Big| (V_x + V_w) \cos \phi - V_Y \sin \phi \Big| \left[Y_{bu}' - r_c \cos(\psi - \gamma)\right] \Big\} - \Big\{ f_3 \left[(V_x + V_w) \cos(\theta + \phi) - V_Y \sin(\theta + \phi)\right]$$

$$\Big| (V_x + V_w) \cos(\theta + \phi) - V_Y \sin(\theta + \phi) \Big| \left[r_4 - r_c \cos(\theta + \gamma - \psi)\right] \Big\} \Bigg] + \Bigg[\frac{dI_1}{dw} \dot{w} + 2 M_1 \Bigg\{ \frac{M_2^2 r_1 r_2 \dot{\theta} \sin(\theta + \gamma)}{M_c^2} -$$

$$M_2^2 \frac{\dot{w}\left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)\right]}{g M_c^3} \Bigg\} + 2 M_2 \Bigg\{ \frac{M_1^2 r_1 r_2 \dot{\theta} \sin(\theta + \gamma)}{M_c^2} +$$

$$\frac{M_1 M_2 \dot{w} \left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)\right]}{g M_c^3} \Bigg\} + \left(\frac{M_1}{M_c}\right)^2 \frac{\dot{w}\left[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)\right]}{g} \Bigg]$$

$$\left[\frac{r_1 r_2 \dot{\theta} \cos(\theta + \gamma) - r_2^2 \dot{\theta}}{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta + \gamma)} - \dot{\phi} + \frac{d\gamma}{dw} \dot{w} \right] \Bigg\}$$

To describe the motion of the system in the pitch plane, two more equations of motion are required. These are obviously:

$$\sum F_X = M_c \ddot{X}$$

and

$$\sum F_Y = M_c \ddot{Y}$$

The external forces are the thrust, weight, and aerodynamic forces. We may rewrite the above equations as:

$$\ddot{X} = \frac{1}{M_c}\left[F \sin\left(\theta_H + \phi\right) - D_1 \sin\phi - D_2 \cos\phi + D_3 \cos\left(\phi + \theta\right)\right]$$

$$\ddot{Y} = \frac{1}{M_c}\left[F \cos\left(\theta_H + \phi\right) - g M_c - D_1 \cos\phi - D_2 \sin\phi - D_3 \sin\left(\phi + \theta\right)\right]$$

expanding the equations:

$$\begin{aligned}\ddot{X} = \frac{1}{M_c}\Big\{&F \sin\left(\theta_H + \phi\right) - f_1\left[V_Y \cos\phi + \left(V_X + V_W\right)\sin\phi\right]\left|V_Y \cos\phi + \left(V_X + V_W\right)\sin\phi\right|\left(\sin\phi\right) - \\ &f_2\left[\left(V_X + V_W\right)\cos\phi - V_Y \sin\phi\right]\left|\left(V_X + V_W\right)\cos\phi - V_Y \sin\phi\right|\left(\cos\phi\right) + f_3\Big[-\left(V_X + V_W\right)\cos\left(\theta + \phi\right) + \\ &V_Y \sin\left(\theta + \phi\right)\Big]\left|-\left(V_X + V_W\right)\cos\left(\theta + \phi\right) + V_Y \sin\left(\theta + \phi\right)\right|\left[\cos\left(\theta + \phi\right)\right]\Big\}\end{aligned}$$

$$\begin{aligned}\ddot{Y} = \frac{1}{M_c}\Big\{&F \cos\left(\theta_H + \phi\right) - f_1\left[V_Y \cos\phi + \left(V_X + V_W\right)\sin\phi\right]\left|V_Y \cos\phi + \left(V_X + V_W\right)\sin\phi\right|\left(\cos\phi\right) - M_c g - \\ &f_2\left[\left(V_X + V_W\right)\cos\phi - V_Y \sin\phi\right]\left|\left(V_X + V_W\right)\cos\phi - V_Y \sin\phi\right|\left(\sin\phi\right) - f_3\Big[-\left(V_X + V_W\right)\cos\left(\theta + \phi\right) + \\ &V_Y \sin\left(\theta + \phi\right)\Big]\left|-\left(V_X + V_W\right)\cos\left(\theta + \phi\right) + V_Y \sin\left(\theta + \phi\right)\right|\left[\sin\left(\theta + \phi\right)\right]\Big\}\end{aligned}$$

The five equations for $\ddot{\phi}$, $\ddot{x}$, $\ddot{y}$, ψ, and r_c may be solved simultaneously to completely describe the motion of the system in the pitch plane.

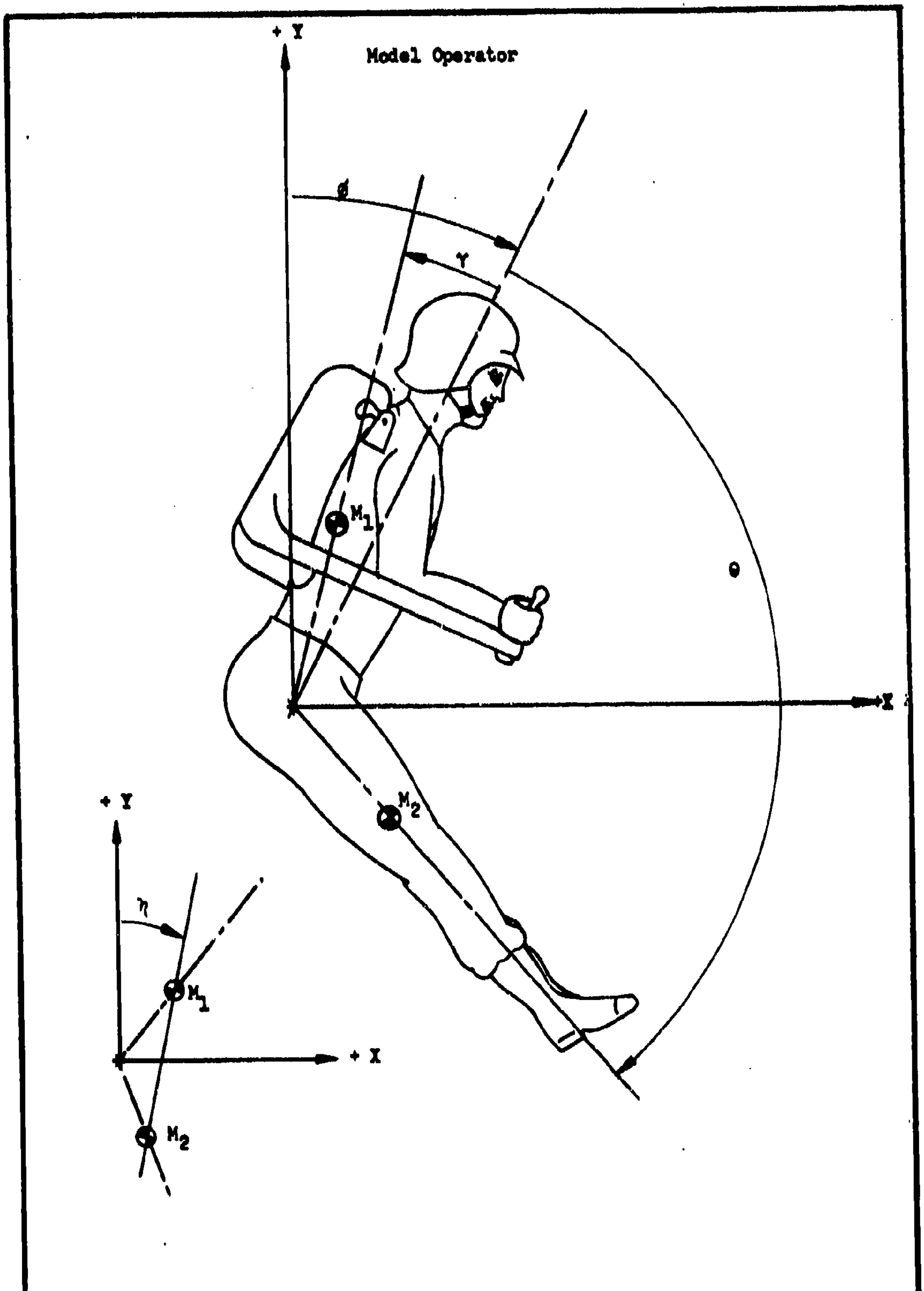


Figure 1
Appendix D

Body Geometry During
Kick Maneuver

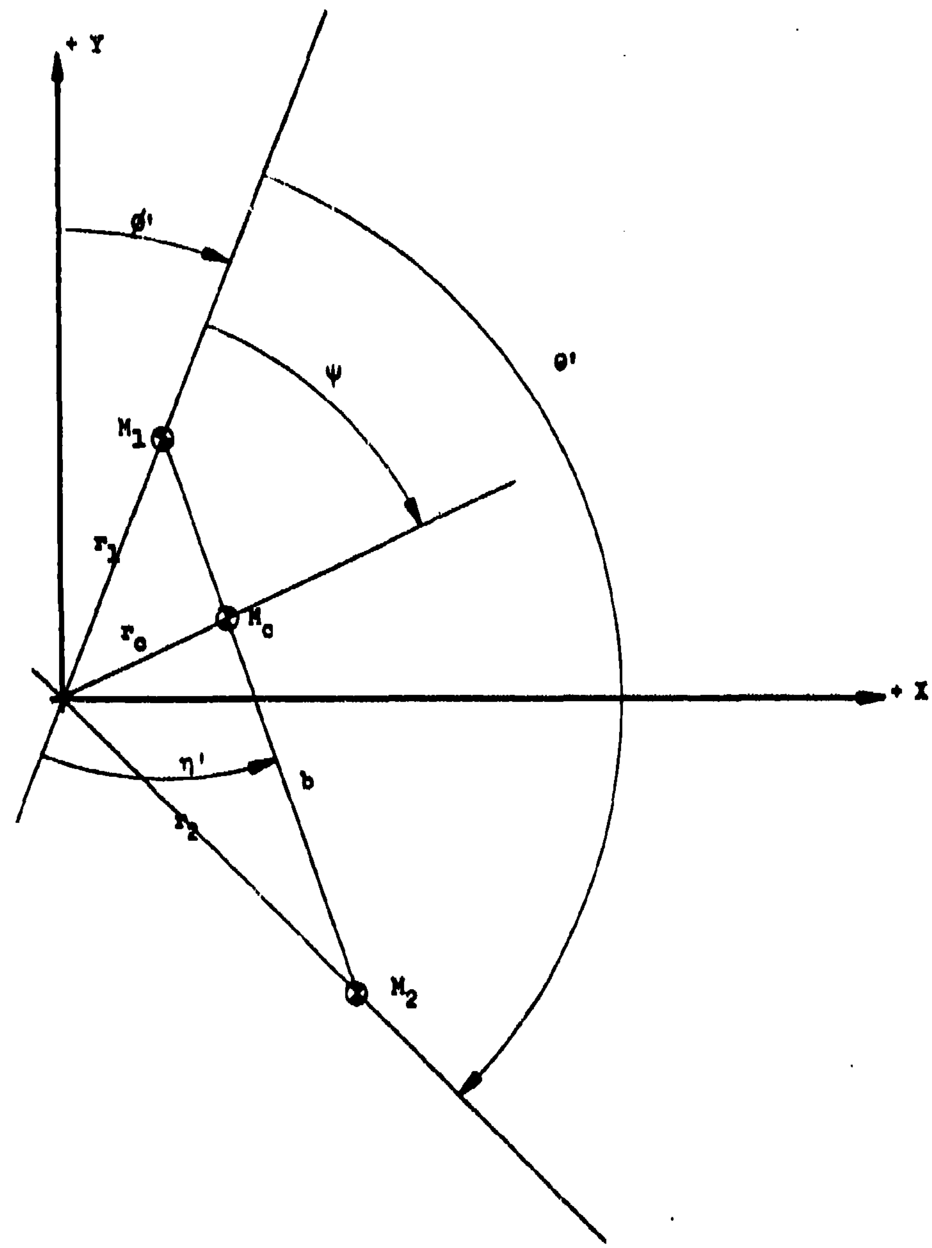


Figure 2
Appendix D

Orientation of Thrust Vector to Model

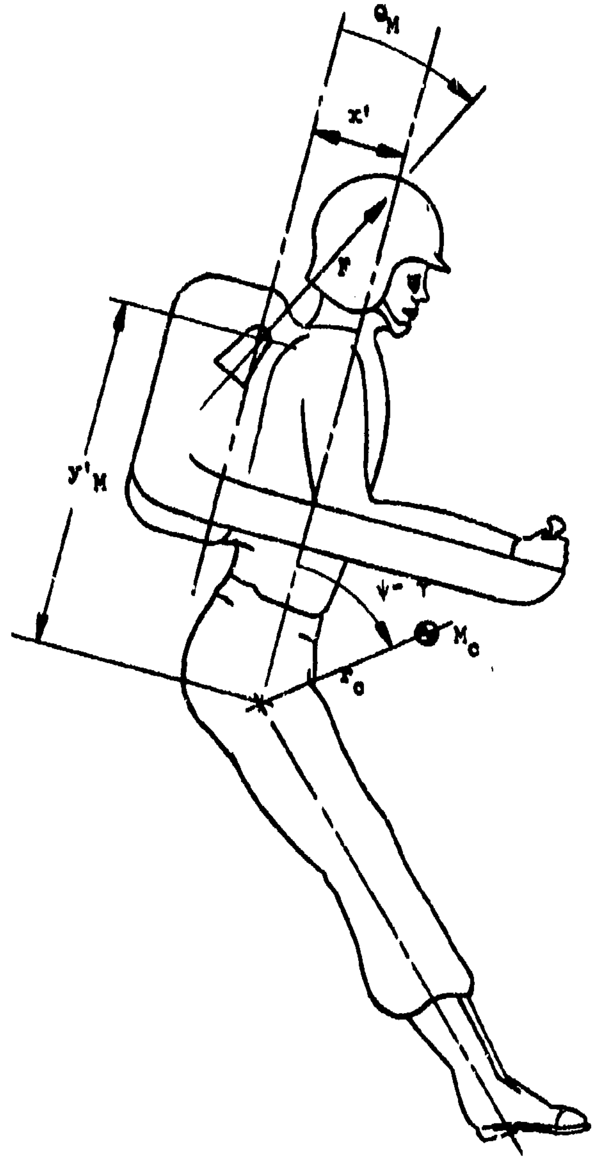


Figure 3
Appendix D

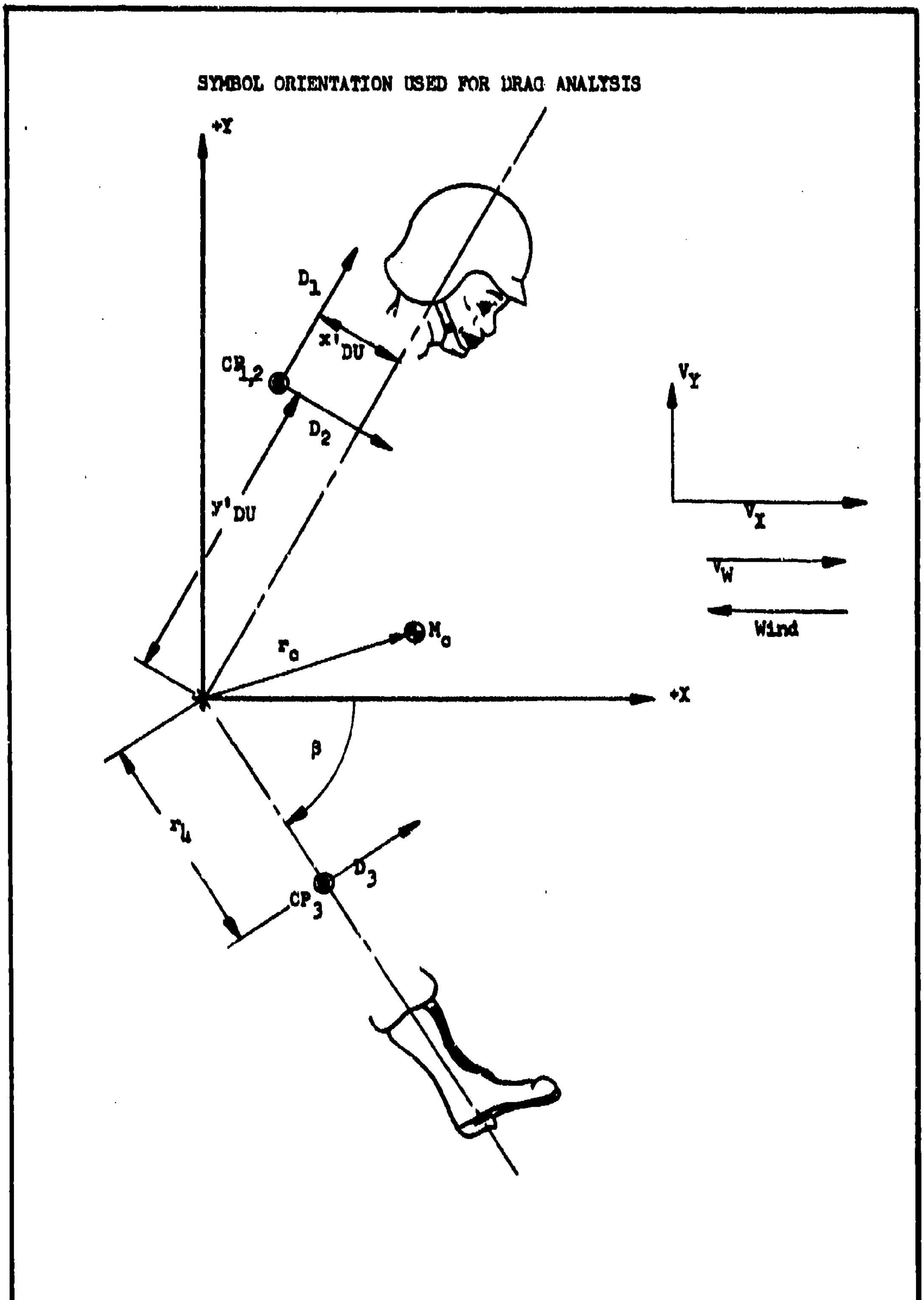


Figure 4
Appendix D

Orientation of Torque Due to Aerodynamic Forces on Model

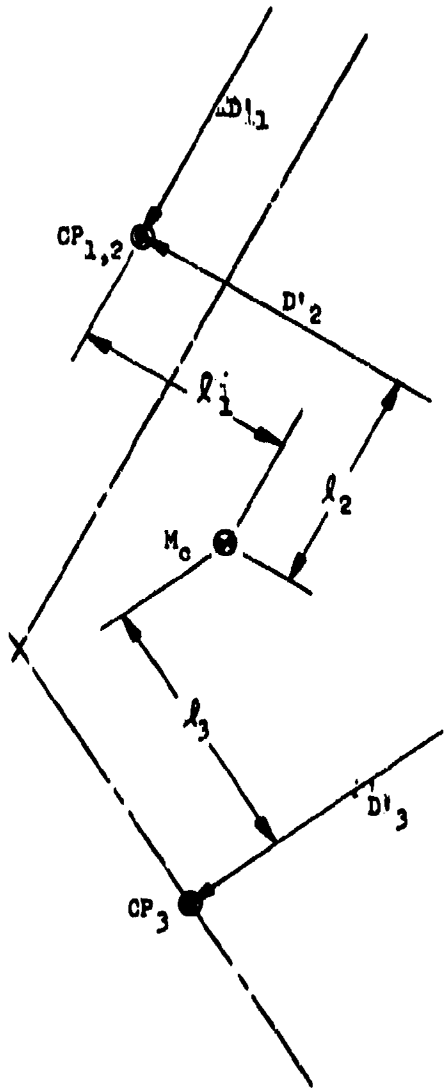


Figure 5
Appendix D